

AD-A172 559 USING THE MONTE CARLO METHOD TO SOLVE PROBLEMS OF THE 1/3
TRANSFER OF FAST EL (U) FOREIGN TECHNOLOGY DIV
WRIGHT-PATTERSON AFB OH A F AKKERMAN ET AL 84 SEP 86
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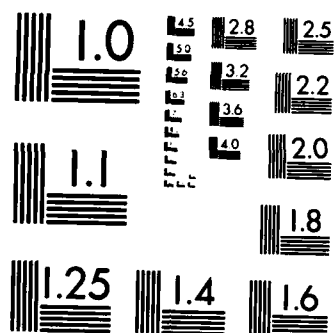
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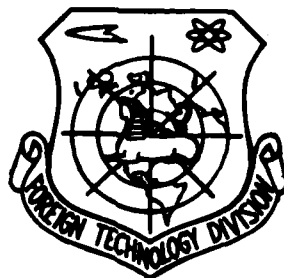
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USING THE MONTE CARLO METHOD TO SOLVE PROBLEMS OF THE TRANSFER OF FAST ELECTRONS IN A SUBSTANCE

by

A.F. Akkerman, Yu. M. Nikitushev, V.A. Botvin



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By: A.F. Akkerman, Yu. M. Nikitushev, V.A. Botvin

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Foreign page numbers occur in the English text and may be found anywhere along the left margin of the page as in this example:

In them occurs the state named "night blindness" - hemeralopia, which, according to the current point of view, is a result of damage of the rod-shaped apparatus of the eye.

Page 51.

However, in recent years it has been shown that with the hereditary pigment degenerations in animals the biochemical changes are observed in all cellular elements of the retina.

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U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	<i>А а</i>	A, a	Р р	<i>Р р</i>	R, r
Б б	<i>Б б</i>	B, b	С с	<i>С с</i>	S, s
В в	<i>В в</i>	V, v	Т т	<i>Т т</i>	T, t
Г г	<i>Г г</i>	G, g	У у	<i>У у</i>	U, u
Д д	<i>Д д</i>	D, d	Ф ф	<i>Ф ф</i>	F, f
Е е	<i>Е е</i>	Ye, ye; E, e*	Х х	<i>Х х</i>	Kh, kh
Ж ж	<i>Ж ж</i>	Zh, zh	Ц ц	<i>Ц ц</i>	Ts, ts
З з	<i>З з</i>	Z, z	Ч ч	<i>Ч ч</i>	Ch, ch
И и	<i>И и</i>	I, i	Ш ш	<i>Ш ш</i>	Sh, sh
Й й	<i>Й й</i>	Y, y	Щ щ	<i>Щ щ</i>	Shch, shch
К к	<i>К к</i>	K, k	Ъ ъ	<i>Ъ ъ</i>	"
Л л	<i>Л л</i>	L, l	Ы ы	<i>Ы ы</i>	Y, y
М м	<i>М м</i>	M, m	Ь ь	<i>Ь ь</i>	'
Н н	<i>Н н</i>	N, n	Э э	<i>Э э</i>	E, e
О о	<i>О о</i>	O, o	Ю ю	<i>Ю ю</i>	Yu, yu
П п	<i>П п</i>	P, p	Я я	<i>Я я</i>	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ë in Russian, transliterate as yě or ě.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	sinh ⁻¹
cos	cos	ch	cosh	arc ch	cosh ⁻¹
tg	tan	th	tanh	arc th	tanh ⁻¹
ctg	cot	cth	coth	arc cth	coth ⁻¹
sec	sec	sch	sech	arc sch	sech ⁻¹
cosec	csc	csch	csch	arc csch	csch ⁻¹

Russian English

rot curl

lg log

GRAPHICS DISCLAIMER

All figures, graphics, tables, equations, etc. merged into this translation were extracted from the best quality copy available.

USING THE MONTE CARLO METHOD TO SOLVE PROBLEMS OF THE TRANSFER OF FAST ELECTRONS IN A SUBSTANCE.

A. F. Akkerman, Yu. M. Nikitushev, V. A. Botvin.

Page 2.

All theoretical and experimental data about interaction of electron with atom necessary for conducting calculation are in detail examined. The methods of the optimum organization of the algorithms of calculations are described. Are given the characteristics of programs for computers, and also the results of calculations, obtained with their aid, in the form, convenient for the practical use during the solution of all possible problems of applied character. Calculations are done for 8 elements and substances with the atomic number from 3.3 to 82 in the wide region of wave energy (0.4-6.0 MeV) and angles of incidence of electrons (0-81°), which makes it possible to carry out reliable interpolation and to use the obtained results for any elements and complicated substances.

Consecutive and systematic presentation of material makes book of available for wide circle of specialists, who work in field of application of electron accelerators for practical purposes, engineers and technologists, connected with construction of different isotopic instruments, and also for students of old policies of VUZ [Institute of Higher Education].

Page 3.

INTRODUCTION.

Transport problem of charged/loaded particles (electrons and positrons) in substance for period of several decades draws interests of researchers. So prolonged an interest in this problem is explained by a large quantity of problems, which arose recently in the new branches of science (space technology, radiation chemistry technology, nuclear engineering, etc.). Experimental research in these regions is conducted by broad front and contains the enormous range of energies of particles, beginning from several keV and ending with tens of GeV. For obtaining the necessary information usually it is necessary to carry out several separate experiments, to use precision instruments, which is natural, it requires large efforts/forces and significant financial expenditures.

Much was done recently in direction of theoretical description of process of transfer of particles through substance. Now the theory, based on the solution of the kinetic equation of transfer, although it is sufficiently perfect from a mathematical point of view, it can, strictly speaking, to be used only for the infinite homogeneous media. Account in this theory even of the simplest boundaries is complex problem. Theory unable to correctly consider fluctuations in the energy losses of particles and does not give their energy and angular distribution.

Large difficulties appear during attempt at consecutive account in equation of transfer of secondary radiation: δ -electrons, bremsstrahlung or products of nuclear interactions.

All these difficulties led to creation of whole series of approximation methods of calculation. However, the essential simplifications, utilized in them, lead to the noticeable divergence of calculations from the results of experiment, which does not always succeed in eliminating by the selection of semi-empirical constants or relationships/ratios. But the difficulties, connected with the consideration of complex boundaries, still remain in this case.

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Thus, was explained that there is class of problems, such, as calculation of passage of particles through protection of complex form, contribution of secondary radiation after protection, energy and angular particle distributions, backscattering, dependence of probability of forming interstitial-vacancy pairs as functions of depth of penetration of particles, obtaining spatial distribution of absorbed in substance energy, which cannot be correctly solved by existing theory or its approximate modifications, but experimental methods they are excessively labor-consuming. The alternative method of the solution of problems by the radiation transfer is the method of Monte Carlo (method for random testing). This method has already long ago been successfully used for the calculations of the passage of

neutrons and γ -quanta through the substance.

Application of method to calculation of passage of charged/loading particles and, in particular, electrons was not so/such intense. Is explained this fact by the large interaction cross section of the charged/loading particles with the substance, as a result of which for obtaining the data even with 10% accuracy necessary to fulfill 10^7 - 10^8 random tests, which virtually cannot be realized on the contemporary computers.

However, when into diagram of Monte Carlo method was introduced series/row of improvements, time for conducting calculation noticeably decreased, and advantages of method - sufficiently common algorithm of calculation, automatic account of boundary conditions, even most complicated, largest volume of information, obtained in one cycle of calculations, and others - select it in comparison with others by Monte Carlo method, it can be done sufficiently high and in many instances, compared with attained in experiment, if necessary accuracy in initial data, that characterize elementary report/event of interaction is provided. For the electrons with the energy to 10 MeV this condition, as a rule is satisfied.

Examination of different aspects of application of Monte Carlo method for solving of those formulated above problems composes content of this work. We did not approach the consecutive presentation of the essence of the Monte Carlo method. At present there are several

excellent monographs, dedicated to this question.

Page 5.

The target, by which we were guided, first, to let to reader fundamental information for conducting such calculations, including initial data, tested by different authors versions of networks, recommendation regarding the organization of programs and different methods of accelerating the calculation. Secondly, so to systematize the results of our calculations so that them it would be possible to use in practice for all possible evaluations/estimates during the engineering developments, connected with the application of electron beams, isotopic sources, etc. Probably, the results of calculations will be useful for researchers, who work in different regions of physics.

All calculations are carried out on computers BESM-3, BESM-4 for series of elements of periodic system in interval of energy of 0.4-6 MeV, which is of greatest interest for practice. The mean time, necessary for conducting one cycle of calculations, is 40-60 min. Even with such relatively larger expenditures of time (during the use of the more advanced machines, for example BESM- 6, this time it can

be reduced almost by an order) economic effectiveness of calculations in comparison with the realization of the corresponding experiments is indisputable.

Rapid increase in number of works, carried out by Monte Carlo method in USSR and abroad, is best confirmation of this position.

Authors express appreciation to academician of the AN [Academy of Sciences] of KazSSSR Zh S. Takibayev for the constant attention with the fulfillment of this work.

Page 6.

The principal notations, utilized in the book, and the value of certain constants.

$\beta = v/c$ - velocity of electron.

$P = (m_0 \beta c) / \sqrt{1 - \beta^2}$ - relativistic momentum of an electron.

$p = \beta / \sqrt{1 - \beta^2}$ - relativistic momentum of an electron in ones $m_0 c$.

E - kinetic energy of electron.

ϵ - kinetic energy of electron in ones $m_0 c^2$.

$\epsilon + 1$ - electronic energy in ones $m_0 c^2$.

k - photon energy in ones $m_0 c^2$.

Z - atomic number of element.

n_0 - number of atoms in 1 cm³ of substance.

$\alpha = Z/137$.

$\Gamma(x)$ - gamma function.

$N = 6.02 \cdot 10^{23}$ mole⁻¹ - Avogadro number.

$m_0 = 9.1 \cdot 10^{-28}$ g - mass of electron.

$e = 4.8 \cdot 10^{-10}$ cgs esu - electron charge.

$c = 3 \cdot 10^{10}$ cm/s - speed of light.

$r_0 = e^2 / (m_0 c^2) = 2.82 \cdot 10^{-13}$ cm - a classical radius of electron.

$h = 6.625 \cdot 10^{-27}$ erg·s - Planck's constant.

$m_0 c^2 = 0.511$ MeV - rest energy of electron.

$\Phi = Z^2 r_0^2 / 137 = 5.78 \cdot 10^{-28} Z^2$ cm².

$\alpha_0 = h^2 / (4\pi^2 m_0 e^2) = 0.529 \cdot 10^{-8}$ cm - radius of the first Bohr orbit.

Page 7.

Chapter I.

CHARACTERISTICS OF ELEMENTARY EVENT.

In this chapter we will consider characteristics elementary event of interaction of electron with atom, which lie at base of calculations by Monte Carlo. Are investigated the field of applicability, agreement with the experiment of different formulas, which describe elastic electron-nuclear, electron-electron interaction, and also inelastic process of braking electron emission.

§ 1. Elastic electron-nuclear scattering.

Differential cross section of scattering in solid angle of $d\Omega = 2\pi \sin \theta d\theta$ electron in nuclear field with pure/clean Coulomb potential Ze/r is described by Rutherford's formula

$$\frac{d\sigma_R}{d\Omega} = \frac{r_0^2 Z^2}{p^4 \beta^4} \frac{1}{(1 - \cos \theta)^2}, \quad (1.1)$$

where p - relativistic momentum of an electron in ones m.c. Formula (1.1) corresponds to Born approximation, and in it the effects, caused by electron spin, are not considered. In this same approximation/approach the difference in scattering of electron and positron is not exhibited.

More complete examination of electron scattering was carried out

by Mott [1]. Using Dirac's equation, he succeeded in obtaining exact expression for the section in the form of the weakly converging series

$$\frac{d\sigma_M}{d\Omega} = \frac{d\sigma_R}{d\Omega} \left\{ \frac{4 \sin^2 \frac{\theta}{2}}{(s+1)^2} |F|^2 + \frac{|G|^2}{s^2} \frac{4p^2 \sin^2 \frac{\theta}{2}}{(s+1)^2 \cos^2 \frac{\theta}{2}} \right\}, \quad (1.2)$$

where $F = F_0 + F_1$ and $G = G_0 + G_1$

$$F_0 = \frac{i\Gamma \left(1 - \frac{i\alpha}{\beta}\right)}{2\Gamma \left(1 + \frac{i\alpha}{\beta}\right)} \exp \left[\frac{i\alpha}{\beta} \ln \sin^2 \frac{\theta}{2} \right],$$

$$F_1 = \frac{1}{2} i \sum_{l=0}^{\infty} [l D_l + (l+1) D_{l+1}] (-1)^l P_l(\cos \theta),$$

$$G_0 = -i \frac{\alpha}{\beta} \left[\operatorname{ctg}^2 \frac{\theta}{2} \right] F_0,$$

$$G_1 = \frac{1}{2} i \sum_{l=0}^{\infty} [l^2 D_l - (l+1)^2 D_{l+1}] (-1)^l P_l(\cos \theta),$$

$$D_l = \frac{\exp(-i\pi l) \Gamma \left(l - \frac{i\alpha}{\beta}\right)}{\left(l + \frac{i\alpha}{\beta}\right) \Gamma \left(l + \frac{i\alpha}{\beta}\right)} = \frac{\exp[-i\pi(l^2 - \alpha^2)^{1/2}] \Gamma \left[(l^2 - \alpha^2)^{1/2} - \frac{i\alpha}{\beta}\right]}{\left[(l^2 - \alpha^2)^{1/2} + \frac{i\alpha}{\beta}\right] \Gamma \left[(l^2 - \alpha^2)^{1/2} + \frac{i\alpha}{\beta}\right]},$$

ϵ , p , β - the energy, pulse/momentum and the velocity of electron before the scattering.

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Thus, expression, which stands in curly braces of formula (1.2), describes relation of sections of Mott and Rutherford, i.e., $R = \frac{d\sigma_M}{d\Omega} / \frac{d\sigma_R}{d\Omega}$. The detailed calculations of this relation as the functions of the scattering angle by θ , atomic number Z and energy of electron E were carried out on computer by Doggett and Spencer [2], and their results for the electrons were given in Table 1. Applications/appendices. Analogous calculation was carried out by

Sherman [3].

Special feature of formula (1.2) is such, that scattering cross section to large angles, close to 180° , requires addition of very large number of terms of series/row and, consequently, data of table in this region of angles can be imprecise.

Bartlett and Watson [4] obtained from (1.2) approximation/approach, valid for small scattering angles

$$R = \frac{d\sigma_M/d\Omega}{d\sigma_R/d\Omega} = 1 + \frac{\pi}{\sqrt{2}} \alpha \beta \cos \gamma (1 - \cos \theta)^{-1}, \quad (1.3)$$

where

$$\cos \gamma = \operatorname{Re} \left\{ \frac{\Gamma\left(\frac{1}{2} - i\alpha\beta\right) \Gamma(1 + i\alpha\beta)}{\Gamma\left(\frac{1}{2} + i\alpha\beta\right) \Gamma(1 - i\alpha\beta)} \right\}. \quad (1.4)$$

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The numerical values $\cos \gamma$ are given in Table 2 of applications/appendices.

Convenient approximation for calculation R is given to McKinley and Feshbach [5]

$$R = 1 - \frac{1}{2} \beta^2 \sin^2 \frac{\theta}{2} + \pi \alpha \beta \left(1 - \sin \frac{\theta}{2} \right) \sin \frac{\theta}{2}. \quad (1.5)$$

Since formula (1.5) is obtained during solution of a precise section

in the series/row according to degrees α (terms, which contain α^n , $n > 3$ they are thrown/rejected), it is applicable only for $\alpha \ll 1$, i.e., for small Z . For the rough estimates the formula can be used up to $Z=30$. As showed checkout calculations, values R for carbon in this formula coincide with the accurate results of Doggett and Spencer.

Given above formulas do not consider ^{the role of} ~~the~~ shadowing of coulomb field of kernel by atomic electrons.

First calculation of shadowing was carried out in Born approximation of Bethe. The tabulated values of the shielded section of Rutherford are given in work [6]. According to Goudsmit and Saunderson [7], the modified section of Rutherford taking into account the parameter of shadowing η takes the form

$$d\sigma = d\sigma_R \left(\frac{1}{1 + \frac{\cos^2 \theta}{2\eta}} \right), \quad (1.6)$$

where $\eta = \theta_1^2/4 = (hZ)^2 / (8\pi\alpha_0 P)$, θ_1 - the so-called angle of shadowing.

As can be seen from formula (1.6), effect of shadowing to become essential only in very small angles, moreover scattering cross section is reduced.

Moliere [8], using WKB [Wentzel-Kramers-Brouillin] approach, obtained expression for calculating parameter of shadowing:

$$\eta_{LM} = 1.7 \cdot 10^{-5} Z^2 \frac{1 - \beta^2}{\beta^4} \left[1.13 + 3.76 \frac{\beta^2}{\beta^2} \right]. \quad (1.7)$$

Nigam et al [9] made more precise Moliere's theory for multiple scattering and for the parameter η they give the following expression

$$\eta_N = 1.7 \cdot 10^{-6} \frac{1-\beta^2}{\beta^3} Z^{1/2} \left\{ 1 + 4\alpha\chi_0 \left[\frac{1-\beta^2}{\beta} \ln \chi_0 + \frac{0.281}{\beta} + 1.448\beta \right] \right\}, \quad (1.8)$$

where $\chi_0 = 0.00825(\sqrt{1-\beta^2}/\beta)Z^{1/2}$.

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As can be seen from Table 1, where are given values η_M and η_N for different energies, the first are systematically more than the seconds, although only for large Z and low energies of electrons difference between them is substantial. Convenient in the calculations by Monte Carlo formula for the section proposed Spencer [10]. He spread formula (1.3), which coincides with a precise formula of Mott (1.2) for the small scattering angles, to all angles, after introducing the correction term $H(\theta)$

$$\frac{d\sigma_M}{d\Omega} = \frac{d\sigma_R}{d\Omega} \left[1 + \frac{\pi}{\sqrt{2}} \alpha\beta \cos \gamma (1 - \cos \theta)^{1/2} + H(\theta) \right], \quad (1.9)$$

whence

$$H(\theta) = R - 1 - \frac{\pi}{\sqrt{2}} \alpha\beta \cos \gamma (1 - \cos \theta)^{1/2}. \quad (1.10)$$

Relation $R = \frac{d\sigma_M}{d\Omega} / \frac{d\sigma_R}{d\Omega}$ is assumed to be known from precise calculations, for example, of Doggett and Spencer. The account of shadowing in (1.9) was carried out by Spencer, using the parameter of Moliere's shadowing η_M . Furthermore, it took into account correction for inelastic electron-electron scattering; for this Z , is substituted by

$Z(Z+1)$. Then scattering cross section is registered in the form

$$\frac{d\sigma}{d\Omega} = \frac{2\pi r_0^2}{p^2 \beta^4} \frac{Z(Z+1)(1+\delta)}{(1 - \cos\theta + 2\eta)^2} \left[1 + \frac{\pi}{\sqrt{2}} \alpha \beta \cos \gamma (1 - \cos\theta + 2\eta)^{1/2} + H(\theta) \right], \quad (1.11)$$

where

$$H(\theta) = R \left(\frac{1 - \cos\theta + 2\eta}{1 - \cos\theta} \right)^2 - 1 - \frac{\pi}{\sqrt{2}} \alpha \beta \cos \gamma (1 - \cos\theta + 2\eta)^{1/2}, \quad (1.12)$$

and

$$\delta = [(Z+1) \ln 4\gamma_M]^{-1} \left[u - \ln \left\{ 0.16 Z^{-1} \left(1 + 3.33 \frac{\alpha^2}{\beta^2} \right) \right\} \right]$$

- the so-called factor of Fano,

u - constant, approximately equal to 5.

Table 1. Parameters of shadowing η ; $\eta_M \cdot 10^3$

$\begin{matrix} Z \\ E \end{matrix}$	6	13	14	22	32	50	82
6.0	3.98	6.78	7.16	10.16	14.19	23.37	49.43
4.0	8.30	14.22	15.01	21.31	29.79	49.10	104.03
2.0	27.58	47.80	49.94	70.97	99.46	164.66	351.32
1.0	82.46	141.72	149.69	213.53	301.15	504.94	1097.90
0.7	138.41	238.85	251.84	360.66	511.85	868.93	1923.61
0.4	293.75	508.62	538.00	778.68	1123.75	1909.86	4566.95

 $\eta_N \cdot 10^3$

$\begin{matrix} Z \\ E \end{matrix}$	6	13	14	22	32	50	82
6.0	3.48	5.83	6.13	8.29	10.65	14.38	20.11
4.0	7.30	12.23	12.85	17.39	22.37	30.23	42.35
2.0	24.26	40.68	42.75	57.90	74.54	100.97	142.15
1.0	72.52	121.64	127.84	178.22	228.20	302.84	427.95
0.7	121.63	208.96	214.15	290.36	373.99	507.14	715.97
0.4	257.44	430.90	452.74	611.81	785.54	1068.79	1477.90

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Convenience in expression (1.12) consists in the fact that correction term $H(\theta)$ can be with high accuracy approximated with the help of the same polynomials $1 - \cos \theta + 2\eta$, which enter into formula (1.11):

$$H(\theta) = \sum_{j=1}^n H_j(\theta) (1 - \cos \theta + 2\eta)^j. \quad (1.13)$$

Already for $n=5$ sum (1.13) differs from the precise value of $H(\theta)$ not more than on $\frac{1}{10}\%$.

It must be noted that account of shadowing, as it is conducted by Spencer, is very approximate. Actually, the sections, calculated by

formulas (1.11) and (1.12), in practice do not differ from the appropriate unshielded sections of Doggett, as is evident from Table 2.

Formulas given above can be used also for positrons. For this it is necessary in them to everywhere replace Z by $-Z$. As far as $\cos \gamma$ (1.4) is concerned, the structure of expression is such, that it does not depend on sign Z .

Consecutive examination of shadowing is carried out by Lin [11]. The starting point of this work is Mott's formula (1A-109v) [18] for an arbitrary form of shadowing. Some results of calculating Lin are given in Table 3 and show that the shadowing leads to a decrease of section during the small-angle scattering and an increase in the section during the scattering to large angles. Recently these results were confirmed by the calculation of Buhring [13] for mercury. That - that the shielded section is more than the unshielded for large angles of scattering, especially important during calculations of back-scatter factor.

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Table 2. Relation $R \left(\frac{d^2 M}{d\Omega^2} / \left(\frac{d^2 R}{d\Omega^2} \right) \right)$

E	10 M_{Je} (V)		4 M_{Je} (V)		1 M_{Je} (V)		0.2 M_{Je} (V)		0.05 M_{Je} (V)	
η°	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
$Z = 13$										
30°	0,997	0,998	0,998	0,998	1,001	1,002	1,013	1,014	1,017	1,018
60°	0,838	0,838	0,840	0,840	0,861	0,861	0,943	0,943	0,999	1,000
105°	0,431	0,431	0,438	0,437	0,499	0,500	0,743	0,743	0,928	0,928
150°	0,081	0,081	0,091	0,091	0,187	0,187	0,566	0,566	0,861	0,861
$Z = 29$										
30°	1,094	1,093	1,093	1,093	1,092	1,092	1,082	1,082	1,054	1,054
60°	0,987	0,988	0,988	0,988	1,004	1,005	1,057	1,057	1,076	1,076
105°	0,543	0,543	0,549	0,549	0,610	0,610	0,848	0,848	1,022	1,022
150°	0,105	0,105	0,116	0,116	0,218	0,218	0,626	0,626	0,950	0,950
$Z = 50$										
30°	1,235	1,234	1,234	1,234	1,223	1,223	1,161	1,162	1,060	1,064
60°	1,274	1,274	1,274	1,275	1,277	1,277	1,262	1,262	1,173	1,170
105°	0,801	0,801	0,807	0,807	0,869	0,869	1,101	1,101	1,230	1,231
150°	0,165	0,165	0,178	0,178	0,305	0,305	0,815	0,816	1,221	1,221
$Z = 82$										
30°	1,358	1,364	1,364	1,361	1,315	1,322	1,133	1,145	0,998	0,998
60°	1,918	1,914	1,912	1,908	1,857	1,852	1,555	1,546	1,122	1,117
105°	1,726	1,727	1,731	1,732	1,777	1,778	1,859	1,861	1,614	1,618
150°	0,422	0,421	0,446	0,446	0,683	0,681	1,592	1,592	2,130	2,128

Note. 1 - data of Table 1 of application/appendix; 2 - calculation according to formula (1.12).

Key: (1). MeV.

Table 3. Relation $\frac{d\sigma_{\text{экп}}}{d\sigma_{\text{неэкп}}}$ for Au and Cu.

E_γ $^\circ$	400 КэВ (1)		200 КэВ (1)		100 КэВ (1)		50 КэВ (1)	
	Z=79	Z=29	Z=79	Z=29	Z=79	Z=29	Z=79	Z=29
10	0,771	0,987	0,636	0,939	0,469	0,869	0,307	0,732
30	0,985	1,03	0,802	1,00	0,794	0,986	0,631	0,966
60	0,995	0,981	0,987	1,00	0,957	1,01	0,870	1,02
90	0,997	0,984	1,01	0,999	1,02	1,02	1,00	1,04
120	1,01	0,996	1,03	1,01	1,07	1,04	1,11	1,05
150	1,02	1,03	1,05	1,01	1,11	1,01	1,19	1,05

Key: (1). keV.

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One should note that according to other theoretical evaluations/estimates (Olsen [18]) $\frac{d\sigma_{\text{экп}}}{d\sigma_{\text{неэкп}}} \sim 1$ for $\theta \sim 180^\circ$. It is at present difficult to comprehend these contradictions to theory.

Unfortunately, are absent systematic experimental data over scattering cross sections, especially to large angles. Most complete measurements $R_{\text{экп}} = \frac{d\sigma_{\text{экп}}}{d\sigma_M}$ in the interval of angles of $30-150^\circ$ were carried out by Rainwater and Rester [14]. Figure 1 shows the part of their results for Al, from which it is evident that with an accuracy to 10% experimental sections will be coordinated with the unshielded section of Mott. Remains incomprehensible the systematic divergence $R_{\text{экп}}$ from 1 for the energies $E > 2$ MeV. More precise experiments of Dougal [15] and Kessler and Weichert [16] better will be coordinated with experimental data [16] for Hg with the calculations of Lin and Sherman (unshielded section).

Thus, scarce experiments confirm need for correct account of shadowing.

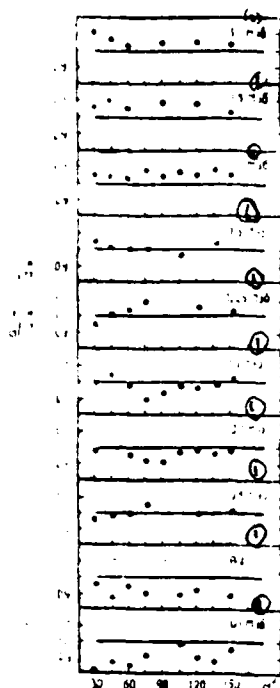


Fig. 1.

Fig. 1. Comparison of experimental elastic-scattering cross sections (point on graphs) with section of Mott for different angles and energies of electrons.

Key: (1). MeV.

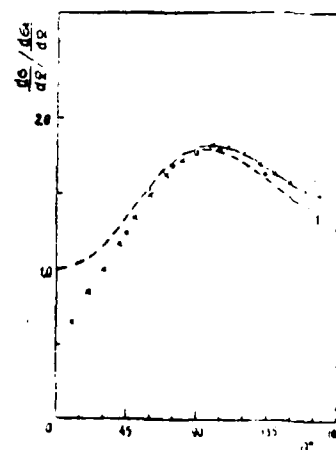


Fig. 2.

Fig. 2. Comparison of values of experimental sections with calculations of Lin (2) and Sherman (1).

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However, the absence of systematic data over the shielded sections makes it necessary to use in the Monte-Carlo calculations the actually unshielded sections. Error introduced in this case will be noticeable at the large scattering angles and with low energies of electrons.

There is one additional source of error in scattering cross section on large angle - radiation effects. As Schwinger [17] showed, the electron, which is diffused in the Coulomb field of the nucleus, with probability 1 emits photons. Therefore in the energy distribution of electrons besides the quasi-elastic peak is present a low-energy "tail", caused by the emission of rigid photons. The theory of this phenomenon is complicated, and its results at present it is possible to examine only as qualitative [18].

Let us define radiation correction ϵ_r as relation

$$\epsilon_r = \int_0^{E_1} \frac{d^2\sigma}{dE_2 d\Omega} dE_2 / \left(\frac{d\sigma}{d\Omega} \right)_0, \quad (1.14)$$

where $\frac{d^2\sigma}{dE_2 d\Omega}$ - double differential section;

E_1 and E_2 - energy of electron before and after scattering;

$(d\sigma/d\Omega)_0$ - section of purely elastic part of scattering for example, section of Mott).

According to Schwinger, for Al and $E_1 = 1$ MeV $\epsilon_r \sim 0.07$, and this value was recently confirmed in experimental work of Berger, et al [19]. In earlier work of Olsen (see [18]), where a radiation

correction is obtained in first Born approximation, it is shown that with energies of electrons $E > 1$ MeV and at scattering angles $\sim 180^\circ$, $\kappa_r > 1$. Figure 3 gives graphs $\kappa_r(E)$ for the different θ .

If, actually, calculations of Olsen are correct, then account of radiation correction can noticeably increase scattering cross section by large angles.

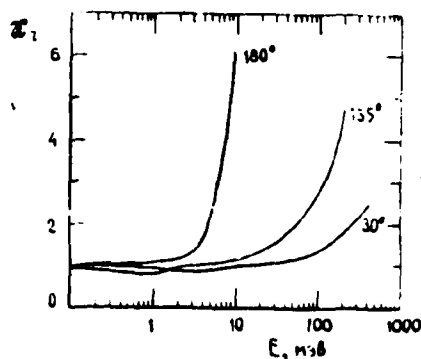


Fig. 3. Graph/diagrams of dependence of radiation correction x_r on energy of electron at different scattering angles.

Page 15. It should be noted that as results of Olsen due to roughness of approximation/approach is not obtained, probably, also essential dependence $x_r(Z)$. However, in the complete scattering cross section radiation correction virtually is not manifested.

Feeding/conducting the result to study of problem about scattering cross sections, one should stress that results of calculations by Monte Carlo contain certain systematic error, caused by inaccuracy of initial theoretical sections.

Further research both theoretical and experimental is necessary, in order to decrease effect of this error.

§ 2. Electron scattering on the electron.

Let us consider kinematics of process elastic-g of electron scattering in free scattering center. Let m_1 - mass of incident

particle (electron), and \vec{P} and \vec{P}_1 - its pulses/momenta before and after collision. Let the mass of the scattering center m_2 - its pulse/momentum after scattering \vec{P}_2 . To the particle scattering m_1 , rested. Then from the laws of conservation of energy and momentum we obtain for the energy, transmitted to recoil particle E_2 (kinetic energy of this particle):

$$E_2 = 2m_2 c^2 \frac{P^2 c^2 \cos^2 \theta}{[m_2 c^2 + (P^2 c^2 + m_1^2 c^4)^{1/2}]^2 - P^2 c^2 \cos^2 \theta}, \quad (1.15)$$

and maximum energy of recoil particle

$$E_{2 \max} = 2m_2 c^2 \frac{P^2 c^2}{m_2^2 c^4 + m_1^2 c^4 + 2m_2 c^2 (P^2 c^2 + m_1^2 c^4)^{1/2}}. \quad (1.16)$$

If scattering center - atom with atomic weight of A , then after small simplifications, connected with neglect of terms, which contain relation $\frac{m_1}{m_2} \ll 1$ and $\frac{E_1 + m_1 c^2}{m_2 c^2} \ll 1$, we will obtain

$$E_{2 \max} = \frac{2E(E + 2m_1 c^2)}{m_2 c^2} = \frac{2188E(E + 1.022)}{A} \text{ eV}, \quad (1.17)$$

where E - energy of electrons in MeV.

Energy, transmitted to recoil atom, is small in comparison with energy, transmitted to electron, and by it are usually overlooked in calculations according to braking of electrons. However, in problems associated with the radiation defects, which appear under the action of fast electrons, the account of energy of efficiency is essential.

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The scattering angles of the particles participating in the collision in the laboratory system of coordinates is calculated with the help of

the following formulas:

$$\cos \theta_1 = \frac{(E_1 + m_1 c^2) (E + (m_1 + m_2) c^2) - (E + m_1 c^2) m_2 c^2 - m_1^2 c^4}{PP_1 c^2} \quad (1.18)$$

and

$$\cos \theta_2 = \frac{[E + (m_1 + m_2) c^2] E_2}{PP_1 c^2}, \quad (1.19)$$

where E_1 and E_2 - energies of particles after scattering.

If two identical particles (two electrons in our case) collide, then after collision it cannot be distinguished, which of electrons is scattered. It is conditionally accepted that the electron with the larger energy is scattered. the maximum energy, devoted by electron to another electron, will be in this case of $E/2$, but not E , as this follows from (1.16) for colliding not the identical particles of the identical mass, for example electron and the positron.

Quantum-mechanical expression for scattering cross section of electron on electron is given by Moller [20] in the form

$$\frac{d^2}{d\omega} = 2\pi r_0^2 \frac{(\epsilon + 1)^2}{\epsilon^2 (\epsilon + 2)} \left[\frac{1}{\omega^2} - \frac{1}{\omega(1-\omega)} \frac{2\epsilon + 1}{(\epsilon + 1)^2} + \frac{1}{(1-\omega)^2} + \frac{\epsilon^2}{(\epsilon + 1)^2} \right], \quad (1.20)$$

where $w\epsilon$ - transmitted energy in units $m_e c^2$ and $0 \leq w \leq 1/2$.

From formulas (1.18) and (1.19) it follows that scattering angle of electron (i.e. electron, which has high energy $\epsilon(i-w)$) is equal to

$$\theta_1 = \arccos \left[\frac{(1-w)(\epsilon+2)}{(1-w)\epsilon+2} \right]^{1/2}, \quad (1.21)$$

but an electron with smaller energy $w\epsilon$ moves at an angle θ_2 to

direction of movement of falling/incident electron

$$\theta_2 = \arccos \left[\frac{w(\epsilon+2)}{w\epsilon+2} \right]^{1/2}. \quad (1.22)$$

In number of cases it is convenient to use section of Moller for scattering in solid angle of $\Omega - \Omega + d\Omega$. Using kinematic relationship/ratio, is easy to obtain expression for $d\sigma/d\Omega$ in the form

$$\frac{d\sigma}{d\Omega} = 4r_0^2 \frac{(\epsilon+1)^2 \cos^2 \theta_1}{\epsilon^2} \left\{ \frac{1}{(\epsilon+2)^2 \sin^4 \theta_1} - \frac{2(2\epsilon+1)}{(\epsilon+1)^2 (\epsilon+2) \sin^2 2\theta_1} - \frac{1}{4 \cos^4 \theta_1} + \frac{\epsilon^2}{(\epsilon+1)^2 (2+\epsilon \sin^2 \theta_1)^2} \right\}. \quad (1.23)$$

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Maximal scattering angle $\theta_{1\max}$ is determined from (1.21) with $w=1/2$ and is equal to

$$\theta_{1\max} = \arccos \left[\frac{\epsilon+2}{\epsilon+4} \right]^{1/2}. \quad (1.24)$$

Let us note that formula (1.20) is valid only for electron scattering on the free quiescent electron ¹.

FOOTNOTE ¹. Moller's formula was generalized in work [21] in the case, when both electrons move. ENDFOOTNOTE.

The presence in bonding electron is usually considered by the introduction to certain threshold they are of a size energy $w'\epsilon$, which corresponds to smallest possible transmission. In principle of $w'\epsilon$ it is possible to identify with the electron-binding energy in the atom,

although for these energies formula (1.20) is yet not applicable.

Total cross section of electron-electron scattering is easy to find, integrating (1.20) in limits of w' and $w = \frac{1}{2}$.

$$\sigma(w) = \int_{w'}^{\frac{1}{2}} \frac{d\sigma}{dw} dw = 2\pi r_0^2 \frac{(\epsilon+1)^2}{\epsilon^2(\epsilon+2)} \left[\frac{1}{w'} - \frac{1}{1-w'} + \left(\frac{\epsilon}{\epsilon+1} \right)^2 \left(\frac{1}{2} - w' \right) - \frac{2\epsilon+1}{(\epsilon+1)^2} \ln \frac{1-w'}{w'} \right], \text{ as } \sigma\left(\frac{1}{2}\right) = 0. \quad (1.25)$$

Knowledge of total cross section e - e - scattering makes it possible to calculate mean free path λ with respect to this scattering

$$\lambda = \frac{1}{\mu} = \frac{1}{n_e Z \sigma(w')}.$$

The process of scattering positrons on electrons was examined by Baba [22], and for section of this scattering expression

$$\begin{aligned} \frac{d\sigma}{dw} = 2\pi r_0^2 \frac{(\epsilon+1)^2}{\epsilon^2(\epsilon+2)w^2} & \left\{ \left[1 - \frac{\epsilon(\epsilon+2)}{(\epsilon+1)^2} w + \frac{1}{2} \left(\frac{\epsilon}{\epsilon+1} \right)^2 w^2 \right] - \right. \\ & \left. - \frac{\epsilon}{\epsilon+2} w^2 \left[\frac{\epsilon+3}{\epsilon+1} - 2 \frac{\epsilon(\epsilon+2)}{(\epsilon+1)^2} w + \left(\frac{\epsilon}{\epsilon+1} \right)^2 w^2 \right] + \right. \\ & \left. + \left(\frac{\epsilon}{\epsilon+2} \right)^2 w^2 \left[\frac{1}{2} + \frac{1}{\epsilon+1} + \frac{3}{2(\epsilon+1)^2} - \left(\frac{\epsilon}{\epsilon+1} \right)^2 w(1-w) \right] \right\}, \quad (1.26) \end{aligned}$$

where $0 \leq w \leq 1$, is obtained.

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§ 3. Braking electron emission.

Inelastic scattering of electrons with emission of bremsstrahlung intensely was studied in recent decades. In one of the most complete surveys/coverage, dedicated to this problem, Koch and Motz [12] give more than 30 formulas for the differential cross sections of bremsstrahlung, obtained in different kind approximations/approaches. The reason for so large a number of formulas limited on the applicability is explained by the impossibility to at present compute the matrix elements of the wave functions of electrons in shielded coulomb field of atom.

Are distinguished two forms of approximate sections of bremsstrahlung: nonrelativistic and relativistic. The first are obtained by Sommerfield [23] and are valid for the electrons from the energies into several keV and for the substance with small Z (shadowing in these sections not taken into consideration). From relativistic region is allotted usually separately a region of super-relativistic ($E > 50$ MeV). The sections of bremsstrahlung for it are given in work [24].

Of greatest interest for the practice are intermediate energies from tens of keV to several ten MeV. The sections of bremsstrahlung in this case are obtained in the first Born approximation, which sets the specific limitations to the applicability of these formulas. They are valid for 1) small Z , 2) far from the maximum energy of braking quantum, 3) speeds $\beta \sim 1$.

However, in spite of these limitations, formulas in first Born

approximation are extensively used even there, where conditions indicated are not satisfied. This is explained by the relative simplicity of the expressions for the section, which make it possible to carry out the qualitative analysis of the obtained experimental results.

Let us consider only several formulas most frequently used in calculations, Monte Carlo (detailed data is presented in survey/coverage [12]).

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Initial in many calculations is the expression for the differential cross section of the bremsstrahlung of Bethe-Heitler [25]:

$$\begin{aligned} \frac{d\Omega}{d\Omega_h d\Omega_p dk} = \frac{Z^2}{137} \left(\frac{r_e}{2\pi} \right)^2 \left[1 - F(q, Z) \right]^2 \frac{p}{k q^4 p_0} \left\{ \frac{p^2 \sin^2 \theta}{(\epsilon + 1 - p \cos \theta)^2} \times \right. \\ \left. \left[4(\epsilon_0 + 1)^2 - q^2 \right] + \frac{p_0^2 \sin^2 \theta_0}{(\epsilon_0 + 1 - p_0 \cos \theta_0)^2} \left[4(\epsilon + 1)^2 - q^2 \right] - \right. \\ \left. - \frac{2pp_0 \sin \theta \sin \theta_0 \cos \Phi [4(\epsilon + 1)(\epsilon_0 + 1) - q^2]}{(\epsilon + 1 - p \cos \theta)(\epsilon_0 + 1 - p_0 \cos \theta_0)} + \right. \\ \left. + \frac{2k^2 (p^2 \sin^2 \theta + p_0^2 \sin^2 \theta_0 - 2pp_0 \sin \theta \sin \theta_0 \cos \Phi)}{(\epsilon + 1 - p \cos \theta)(\epsilon_0 + 1 - p_0 \cos \theta_0)} \right\}, \quad (1.27) \end{aligned}$$

where $\vec{q} = \vec{p}_0 - \vec{p} - \vec{k}$, the pulse/momentum, transmitted to the nucleus, and

$$q^2 = p^2 + p_0^2 + k^2 - 2p_0 k \cos \theta_0 + 2pk \cos \theta - 2pp_0 (\cos \theta \cos \theta_0 + \sin \theta \times \sin \theta_0 \cos \Phi);$$

k and \vec{k} - energy and impulse of photon in units $m_0 c^2$ and $m_0 c$;

ϵ_0, ϵ - initial and final energies of electron;

\vec{p}_0, \vec{p} - initial and final momentum of an electron;

θ_0, θ - angles between \vec{p}_0, \vec{p} and \vec{k} to respectively;

Φ - azimuth angle

$$d\Omega_h = \sin \theta_0 d\theta_0 d\Phi, \quad d\Omega_p = \sin \theta d\theta d\Phi;$$

$F(q, Z)$ - the atomic shape factor, whose numerical values are given in work [8].

Conditions formulated above for applicability of Born approximation are written/recorded with the help of following inequalities

$$\frac{2\pi Z(\epsilon_0+1)}{137 \sqrt{(\epsilon_0+2)\epsilon_0}} \ll 1 \text{ and } \frac{2\pi Z(\epsilon+1)}{137 \sqrt{(\epsilon+2)\epsilon}} \ll 1.$$

Integration of formula (1.27) can be carried out only numerically. However, for case of $F(q, Z)=0$ integration can be carried out analytically, and for the differential cross section of photon emission with the energy in interval of $k-k+dk$ is obtained the expression

$$\begin{aligned} \frac{d\sigma}{dk} = & \frac{Z^2 r_0^2}{137} \frac{p}{k p_0} \left\{ \frac{4}{3} - 2(\epsilon_0+1)(\epsilon+1) \frac{p^2+p_0^2}{p^3 p_0^3} + \frac{\chi_0(\epsilon+1)}{p_0^3} \right. \\ & + \frac{\chi(\epsilon_0+1)}{p^3} - \frac{\chi\chi_0}{p p_0} + L \left[\frac{8(\epsilon_0+1)(\epsilon+1)}{3 p_0 p} + \right. \\ & + \frac{k^3((\epsilon_0+1)^2(\epsilon+1)^2+p_0^2 p^2)}{p_0^3 p^3} + \frac{k}{2 p_0 p} \left(\frac{(\epsilon_0+1)(\epsilon+1)+p_0^2}{p_0^3} \chi_0 - \right. \\ & \left. \left. - \frac{(\epsilon_0+1)(\epsilon+1)+p^2}{p^3} \chi + \frac{2k(\epsilon_0+1)(\epsilon+1)}{p^2 p_0^2} \right) \right] \Bigg\}, \quad (1.28) \end{aligned}$$

where $L = 2 \ln \frac{(\epsilon_0+1)(\epsilon+1)+p_0 p - 1}{k}$; $\chi_0 = 2 \ln(\epsilon_0+1+p_0)$; $\chi = 2 \ln(\epsilon+1+p)$.

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Calculations show that effect of shielding is essential only for low photon energy. Usually to evaluate the possibility of neglect of shadowing, investigate the dimensionless parameter γ , which has the significance of the ratio of the atomic radius in Thomas-Fermi model to the maximum impact parameter

$$\gamma = \frac{100 k Z^{-1/3}}{(\epsilon_0+1)(\epsilon_0+1-k)}. \quad (1.29)$$

From (1.29) it is evident that when $k \rightarrow 0$, $\gamma \rightarrow 0$ and shadowing is substantial, and when $k \rightarrow \infty$, $\gamma \gg 1$ and screening effect is negligible. It is considered that the shadowing can be disregarded/neglected up to the values $\gamma \sim 15$ which makes it possible instead of precise formulas to use maximum expressions for $\gamma \rightarrow \infty$.

Schiff [26] proposed a number of simple analytical expressions for section taking into account shadowing. most general/common of them takes the form

$$\frac{d\sigma}{dk} = \frac{4Z^2 r_0^2}{137k} \left\{ \left[1 + \left(\frac{\epsilon+1}{\epsilon_0+1} \right)^2 \right] \left[\frac{\Phi_1(\gamma)}{4} - \frac{1}{3} \ln Z \right] - \right. \\ \left. - \frac{2}{3} \frac{\epsilon+1}{\epsilon_0+1} \left[\frac{\Phi_2(\gamma)}{4} - \frac{1}{3} \ln Z \right] \right\}. \quad (1.30)$$

In the case of complete shadowing $\gamma=0$, and $\Phi_1=4 \ln 183$ and $\Phi_2=4 \ln 183 - 2/3$, then formula (1.30) passes in

$$\frac{d\sigma}{dk} = \frac{4Z^2 r_0^2}{137k} \left\{ \left[1 + \left(\frac{\epsilon+1}{\epsilon_0+1} \right)^2 \right] - \frac{2}{3} \frac{\epsilon+1}{\epsilon_0+1} \right\} \ln 133Z - \\ + \frac{1}{9} \frac{\epsilon-1}{\epsilon_0+1}. \quad (1.31)$$

For $\gamma < 2$ is used formula (1.30), and values $\Phi_1(\gamma)$ and $\Phi_2(\gamma)$ are removed/taken from graphs of Figure 4.

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For $2 < \gamma < 15$ it is necessary to use expression

$$\begin{aligned}
 \frac{4Z^4 r_0^3}{167k} \left[1 - \left(\frac{\epsilon + 1}{\epsilon_0 + 1} \right)^2 - \frac{2}{3} \frac{\epsilon + 1}{\epsilon_0 + 1} \right] \left[\ln \frac{2(\epsilon_0 + 1)(\epsilon + 1)}{\epsilon} \right. \\
 \left. - \frac{1}{2} - C(\gamma) \right], \quad (1.32)
 \end{aligned}$$

moreover function $C(\gamma)$ is given on graph of Figure 5.

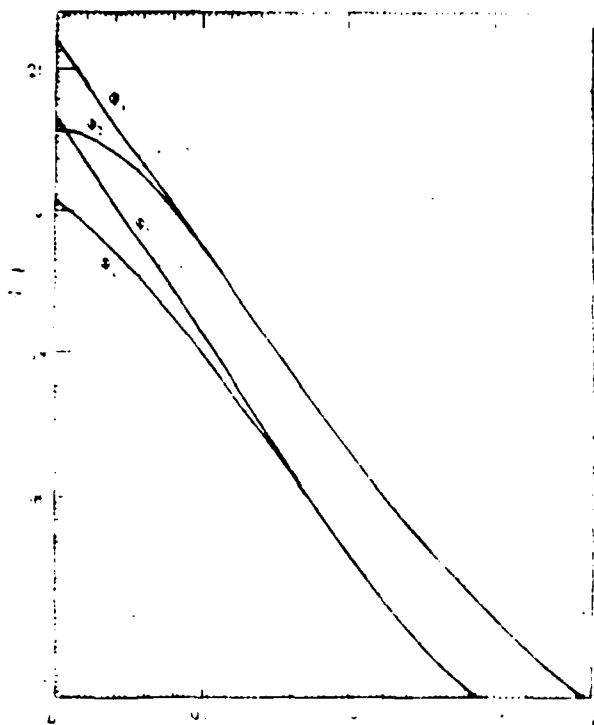


Fig. 4. Plotted functions $\Phi_1(\gamma)$ and $\Phi_2(\gamma)$. 1 - calculation with precise wave functions; 2 - calculation according to Thomas-Fermi model.

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Frequently in region of so-called intermediate shadowing $2 < \gamma < 15$ is used more exact expression of differential cross section, obtained by Bethe [27]

$$\begin{aligned} \frac{d\sigma}{d\Omega} = \frac{2Z^2 r_0^2}{187b} \left\{ \left[1 + \left(\frac{\epsilon+1}{\epsilon_0+1} \right)^2 - \frac{2}{3} \frac{\epsilon+1}{\epsilon_0+1} \right] [\ln M(0) + 1 - \right. \\ \left. - \frac{2}{b} \operatorname{arctg} b] + \frac{\epsilon+1}{\epsilon_0+1} \left[\frac{2}{b^2} \ln(1+b^2) + \right. \right. \\ \left. \left. + \frac{4(2-b^2)}{3b^3} \operatorname{arctg} b - \frac{8}{3b^3} + \frac{2}{9} \right] \right\}, \quad (1.33) \end{aligned}$$

where

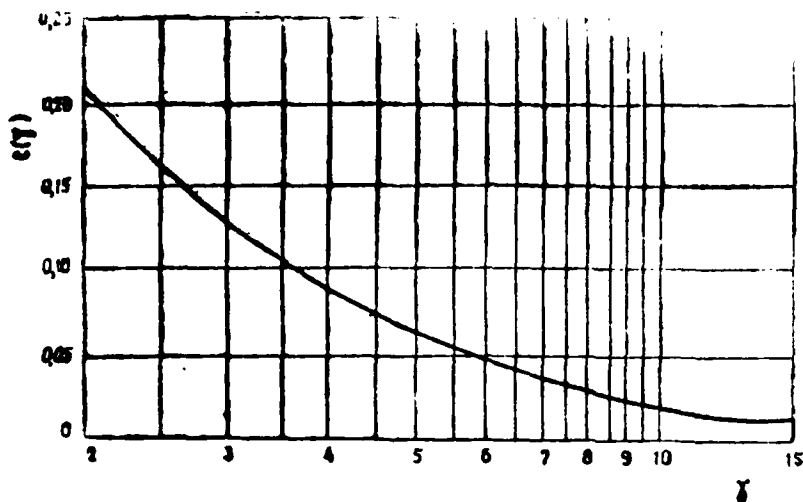
$$b = \frac{2(\varepsilon_0 + 1)(\varepsilon_0 + 1 - k)Z^{1/2}}{111k} = \frac{1}{\gamma}$$

and

$$\frac{1}{M(0)} = \left[\frac{k}{2(\varepsilon_0 + 1)(\varepsilon + 1)} \right]^2 + \left(\frac{Z^{1/2}}{111} \right)^2.$$

In the case of complete absence of shadowing ($\gamma \rightarrow \infty$) both formula (1.33) and (1.32) they lead to one and the same maximum expression

$$\begin{aligned} \frac{d\sigma}{dk} &= \frac{4Z^4 r_0^2}{137k} \left[1 + \left(\frac{\varepsilon + 1}{\varepsilon_0 + 1} \right)^2 - \frac{2}{3} \frac{\varepsilon + 1}{\varepsilon_0 + 1} \right] \times \\ &\times \left[\ln \frac{2(\varepsilon_0 + 1)(\varepsilon + 1)}{k} - \frac{1}{2} \right]. \end{aligned} \quad (1.34)$$

Fig. 5. Plotted function $C(\gamma)$.

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Heitler [28] deduced value Φ , named total cross section and defined as the section of energy loss to the emission

$$\Phi = \frac{1}{\epsilon_0 + 1} \int_0^{\epsilon_0} k \frac{d\sigma}{dk} dk. \quad (1.35)$$

In the approximation/approach, used by Schiff,

$$\Phi = \frac{16}{3} \frac{Z^2 r_0^2}{137}. \quad (1.36)$$

Segre [29] gives the following formula for Φ :

$$\Phi = \frac{4Z^2 r_0^2}{137} \left[\ln 2(\epsilon_0 + 1) - \frac{1}{3} \right]. \quad (1.37)$$

Sections, calculated according to given above formulas, compare with experiment [12]. It is shown that only for the wave energies of electrons $2 \text{ MeV} < E < 10 \text{ MeV}$ the theoretical sections with an accuracy to 10% will be coordinated with the experiment. In extremely

relativistic region of energies these formulas give the overstated values, while in the region of energies < 2 MeV - those understated.

In extremely relativistic region of energies it is possible to remove/take limitation of Born approximation about smallness Z and to obtain expressions for sections, valid for any Z .

We introduce only a few formulas, which are analogs of formulas (1.30)- 1.34),

$$\frac{d\sigma}{dk} = \frac{4Z^2 r_0^2}{137k} \left\{ \left[1 + \left(\frac{\epsilon+1}{\epsilon_0+1} \right)^2 \right] \left[\frac{\Phi_1(\gamma)}{4} - \frac{\ln Z}{3} - f(Z) \right] - \right. \\ \left. - \frac{2}{3} \frac{\epsilon+1}{\epsilon_0+1} \left[\frac{\Phi_2(\gamma)}{4} - \frac{\ln Z}{3} - f(Z) \right] \right\}, \quad (1.38)$$

where $f(Z) = \begin{cases} 1,2021\alpha^2 & \text{для } \overset{(1)}{\text{малых}} Z \\ 0,925\alpha^2 & \text{для } \overset{(2)}{\text{больших}} Z. \end{cases}$

Key: (1). for small Z . (2). for large Z .

For complete shadowing

$$\frac{d\sigma}{dk} = \frac{4Z^2 r_0^2}{137k} \left\{ \left[1 + \left(\frac{\epsilon+1}{\epsilon_0+1} \right)^2 - \frac{2}{3} \frac{\epsilon+1}{\epsilon_0+1} \right] \left[\ln 183Z^{-1/3} - \right. \right. \\ \left. \left. - f(Z) \right] + \frac{1}{9} \frac{\epsilon+1}{\epsilon_0+1} \right\}. \quad (1.39)$$

For $\gamma < 2$ $d\sigma/dk$ is computed from formula (1.38). Functions $\Phi_1(\gamma)$ and $\Phi_2(\gamma)$ are found from the graphs of Figure 4.

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In the interval $2 < \gamma < 15$ formula

$$\frac{d\sigma}{d\Omega} = \frac{4Z^2 r_0^2}{137k} \left[1 + \left(\frac{\epsilon_0 + 1}{\epsilon_0 - 1} \right)^2 - \frac{2}{3} \frac{\epsilon_0 + 1}{\epsilon_0 - 1} \right] \left[\ln \frac{2(\epsilon_0 + 1)(\epsilon_0 - 1)}{k} - \frac{1}{2} - C(\gamma) - f(Z) \right], \quad (1.40)$$

is used and $C(\gamma)$ is found from graph of figure 5.

Expressions for total cross sections take form

$$\Phi = \frac{4Z^2 r_0^2}{137} \left[\ln 183 Z^{-1/2} - \frac{1}{18} - f(Z) \right], \quad (1.41)$$

and

$$\Phi = \frac{4Z^2 r_0^2}{137} \left[\ln 2(\epsilon_0 + 1) - \frac{1}{3} - f(Z) \right]. \quad (1.42)$$

According to evaluations/estimates of Bethe, these formulas ensure accuracy, best, than 2% for energies $E > 50$ MeV and $Z \sim 80$.

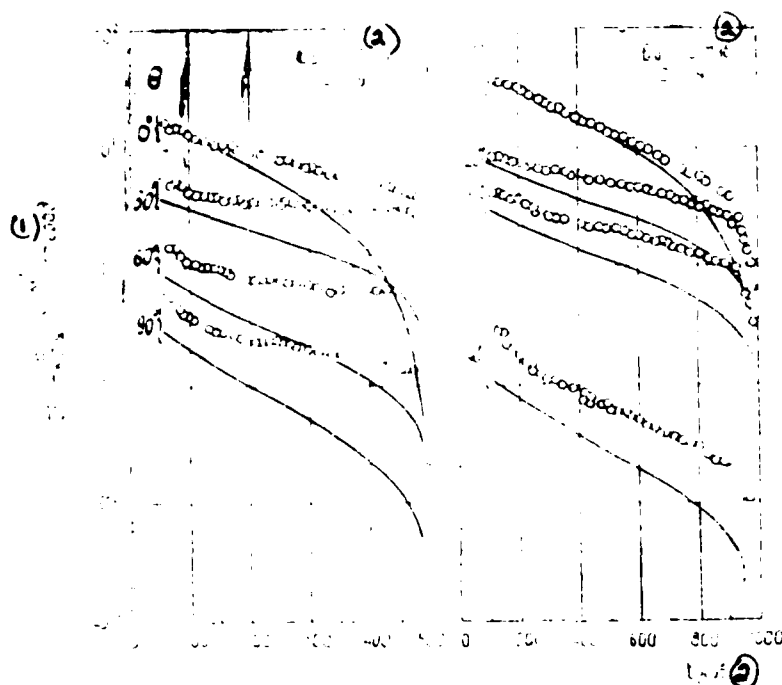


Fig. 6. Comparison of experimental spectra of bremsstrahlung, which emerges at angle θ (small circles), with theoretical (unbroken curves) with different energies of electrons.

Key: (1). sterad. (2). MeV.

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Formulas (1.30)-1.34) can be improved for the purpose of the decrease of the limitation of that superimposed by Born approximation. Usually this is achieved by introduction to formulas (1.30)- 1.34) corrections in the form of the corresponding factors. This correction method of sections assumes that the theory correctly transmits distribution according to the energy of the photons of bremsstrahlung. For the high energies of electrons ($E > 4$ MeV) this, probably, it is correct. For low energies there is a noticeable difference in the form of the

spectrum from the experiment. Difference is especially substantial near the maximum photon energy (Fig. 6).

For nonrelativistic energies of Elwert [30] it proposed correction in the form

$$f_{\text{el}} = \frac{\left[1 - \exp \left\{ -\frac{2\pi Z}{137} \frac{\epsilon_0 + 1}{(\epsilon_0 + 2)\epsilon_0} \right\} \right] (\epsilon_0 + 1) + (\epsilon_0 + 2)\epsilon_0}{\left[1 - \exp \left\{ -\frac{2\pi Z}{137} \frac{\epsilon_0 + 1}{(\epsilon_0 + 2)\epsilon_0} \right\} \right] (\epsilon_0 + 1) + (\epsilon_0 + 2)\epsilon_0} \quad (1.43)$$

According to evaluations/estimates, during use of this correction it is possible to obtain agreement with experiment in limits of 10%, if $E_0 \leq 0.1$ MeV. The region of energies 0.1-2.0 MeV is most difficult for the determination of corrections. Here there are no theoretically obtained corrections whatever and usually they use empirical. They are located as follows. The relation of the total cross sections of empirical (experimental) and Born taking into account shadowing as function ϵ_0 and Z is constructed.

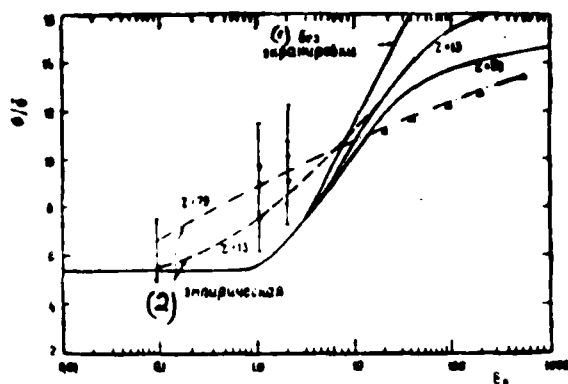


Fig. 7. Empirical curves (broken) of total cross sections of bremsstrahlung. Small circles - results of experiment; triangles - calculation in extremely relativistic region; unbroken curves - theoretical dependences.

Key: (1). Without the shadowing. (2). Empirical.

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Empirical values are obtained by mating of experimental dependence $\Phi_{\text{кст}}(\epsilon_0, Z)$ with theoretical - $\Phi_B(\epsilon_0, Z)$ in extremely relativistic region (Fig. 7). Berger [31] led the calculations of relations $A = \frac{\Phi_{\text{кст}}}{\Phi_B}$ for different Z and energies of electrons. The graph/diagrams of dependence $A(Z, \epsilon_0)$ are shown in Figure 8 and can be used for the interpolation.

As can be seen from curves, correction is maximum with energies $\epsilon_0 = 1.05 \text{ MeV}$ independent of atomic number.

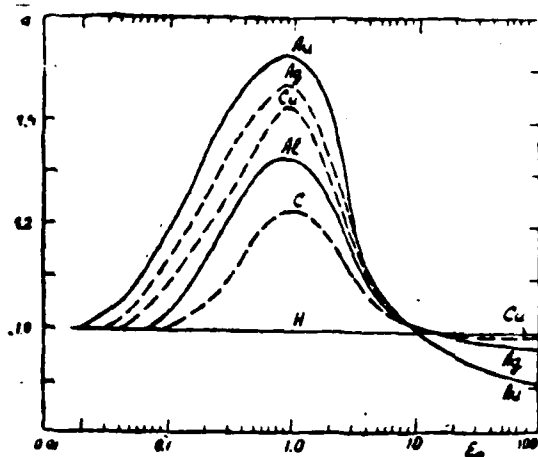
Fig. 8. Graphs of the dependence of $A(Z, \epsilon_0)$.

Table 4. The corrected sections of bremsstrahlung.

(1) Кинетическая энергия электрона E_0 , Мэв	(2) Исправленные сечения $d\sigma$	(3) Неисправленные сечения (номер формулы) σ_1	(4) Область применения	(5) Ожидаемая точность
0,10—2,0	$d\sigma = A f_g d\sigma_1$	1,28	$h > 0,01\epsilon_0$	$\pm 20\%$
2,0—15	$d\sigma = A d\sigma_1$	1,28	$\gamma > 15$	—
	$A d\sigma_1$	1,32	$2 < \gamma < 15$	$\pm 5\%$
	$A d\sigma_1$	1,30	$\gamma < 2$	$\pm 5\%$
15—50	$d\sigma = d\sigma_1$	1,28	$\gamma > 15$	—
	$A d\sigma_1$	1,32	$2 < \gamma < 15$	$\pm 3\%$
	$A d\sigma_1$	1,30	$\gamma < 2$	$\pm 3\%$
50—500	$d\sigma = d\sigma_1$	1,28	$\gamma > 15$	—
	$d\sigma_1$	1,40	$2 < \gamma < 15$	$\pm 3\%$
	$d\sigma_1$	1,38	$\gamma < 2$	$\pm 3\%$

Key: (1). Kinetic energy of electron E_0 , MeV. (2). Corrected sections $d\sigma$. (3). Defective sections (number of formula) σ_1 . (4). Field of application. (5). Expected accuracy.

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In Table 4, borrowed from [12], it is shown, what corrective multipliers it is necessary to use in different energy ranges.

Approximations/approaches, utilized for calculations of section, do not make it possible to consider its behavior near maximum photon energy. In this case, as more correct calculations by Fano, etc. [32] showed, sections were different from zero (in the Born

approximation - 0) and satisfactorily agrees with the experiment.

Since the discussion deals with refinement of section directly at point $k=\epsilon_0$, in calculations by Monte Carlo these corrections can be disregarded/neglected.

In conclusion it is necessary to consider one additional possible channel for emission - electron-electronic interaction. At present there is no theory of this complicated process. Its role, apparently, is unessential against the background considerably of more probable emission as a result of an electron-nuclear interaction. The approximate account of bremsstrahlung in e-e-interaction is produced by replacement in the formulas for the section of term Z^2 on $Z(Z+1)$.

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Chapter II.

PROCESSES OF REPEATED INTERACTION OF ELECTRONS.

Sections of elementary reports/events of interaction of electron with atoms of substance are great, and therefore even in small route segments in substance it tests/experiences large collision frequency. This fact makes it possible with the high accuracy to describe electron motion in the substance, using statistical models. In this chapter different models of multiple scattering are examined, and also a question about the average/mean energy losses to the ionization and the bremsstrahlung are investigated the fluctuations of these losses.

§ 1. Average/mean energy losses of electrons to the ionization and the bremsstrahlung.

Known that average/mean energy loss over a unit of distance due to any process is determined with the help of expression

$$-\frac{dE}{dx} = n_0 \int_{Q_{\min}}^{Q_{\max}} Q \frac{d\sigma}{dQ} dQ, \quad (2.1)$$

where Q - energy, transmitted in single report/event of interaction;

$d\sigma/dQ$ - section of this transmission;

n_0 - number of atoms in 1 cm³ of medium;

Q_{\min} and Q_{\max} - minimum and maximum energy transfers, computed from kinematic relationships/ratios.

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In interesting us case of loss energies to ionization will be registered in the form

$$-\frac{dE}{dx} = m_0 c^2 Z n_0 \varepsilon \int_0^{w_{\max}} w \frac{d\sigma}{dw} dw, \quad (2.2)$$

moreover E is measured in MeV, and path in see integral in formula (2.2) necessary to compute separately for two intervals of $0 < w \leq w'$ and $w' \leq w \leq 1/2$. In the first the electron is considered as connected, and integral, according to Bethe [33], is considered with the help of the expression

$$\int_0^{w'} w \frac{d\sigma}{dw} dw = 2\pi r_0^2 \frac{(\varepsilon+1)^2}{\varepsilon(\varepsilon+2)} \left[\ln \frac{2(m_0 c^2)^2 \varepsilon^2 (\varepsilon+2) w'}{I^2} - \frac{2}{(\varepsilon+1)^2} \right], \quad (2.3)$$

where I - the so-called ionization potential (in more detail about it it will be said below).

In second interval for $d\sigma/dw$ Moller's formula (1.20) is used. Storing/adding up both integrals and taking into account that $w' \ll 1$, we will obtain

$$\frac{dE}{dx} = 2\pi r_0^2 m_0 c^2 Z n_0 \frac{(\varepsilon+1)^2}{\varepsilon(\varepsilon+2)} \left[\ln \frac{m_0^2 c^4 \varepsilon^2 (\varepsilon+2)}{2I^2} + f^-(\varepsilon) \right], \quad (2.4)$$

where

$$f^-(\varepsilon) = \frac{1}{(\varepsilon+1)^2} + \frac{1}{8} \left(\frac{\varepsilon}{\varepsilon+1} \right)^2 - \frac{2\varepsilon+1}{(\varepsilon+1)^2} \ln 2. \quad (2.5)$$

Frequently formula (2.4) is called Bethe-Bloch formula.

For positrons during calculation of second integral is used

formula (1.26) of Bhabha [22]. Expression for $-dE/dx$ is obtained analogous (2.4), but instead of $f(\epsilon)$ it is necessary to substitute the expression

$$f^-(\epsilon) = 2 \ln 2 - \frac{(\epsilon+2)}{12(\epsilon+1)^2} \left[23 + \frac{14}{\epsilon+2} - \frac{10}{(\epsilon+2)^2} + \frac{4}{(\epsilon+2)^3} \right]. \quad (2.6)$$

In formulas examined was not considered polarization of medium, which appears during interaction of electron with atoms. This phenomenon known even by the name of the "effect of density" leads to the decrease of average/mean ionization losses.

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Formula for calculating the average/mean ionization losses and excitation taking into account the effect of density is written/recorded in the form

$$-\frac{1}{\rho} \frac{dE}{dx} = 2\pi r_0^2 \frac{NZ}{A} m_0 c^2 \left[\ln \frac{(m_0 c^2)^2 \epsilon^2 (\epsilon+2)}{2I^2} f^-(\epsilon) - \delta \right]. \quad (2.7)$$

In this formula energy losses are expressed in $\text{MeV} \cdot \text{cm}^2/\text{g}$.

Correction for effect of density was computed in work of Sternheimer [34]. The formulas obtained by him for δ take the form

$$\begin{aligned} \delta &= 4.606x + c + a(x_1 - x)^m & x_0 < x < x_1 \\ \delta &= 4.606x + c & x > x_1 \\ \delta &= 0 & x < x_0, \end{aligned} \quad (2.8)$$

where $x = \lg \rho$; x_0 , x_1 , m , a , c - constants, whose values for different substances are given in Table 5.

From Table 5 it follows that for energy $E_0 \leq 1$ MeV effect of density can be not at all considered.

Ionization potential I is important constant, necessary for calculations according to formula (2.7). It cannot be accurately calculated by contemporary theory, with exception of hydrogen and helium.

According to Bloch, for heavy elements it is possible to use approximation formula

$$I \approx 13,5 Z. \quad (2.9)$$

However, the experimentally obtained values most frequently are used. The systematic measurements I are taken by Bakker and Segre [35], and also Golovin et al. [36]. These data are cited in Table 5.

Table 5.

(1) Материал	I, m (2)		x_1	x_0	c	a	m
	[35]	[36]					
Li	34	—	2	-0,05	3,07	0,374	3,06
Be	60,4	64	2	-0,10	2,83	0,413	2,82
C	76,4	80	2	-0,05	3,22	0,531	2,63
Al	150	163	3	0,05	4,21	0,0908	3,51
Fe	243	—	3	0,10	4,62	0,127	3,29
Cu	279	314	3	0,20	4,74	0,119	3,38
Ag	422	—	3	0,20	5,75	0,251	2,88
Sn	472	—	3	0,20	6,28	0,404	2,52
W	680	—	4	0,30	6,03	0,0283	3,91
Pb	737	788	4	0,40	6,93	0,0652	3,41
U	863	—	4	0,30	6,69	0,0652	3,37
(CH ₃) _n	—	82	2	0,12	2,94	0,393	2,86

Key: (1). Material. (2). eV.

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By another source of energy losses fast electron, moving in substance, are losses to bremsstrahlung. Expression for the average/mean losses over a unit of distance in this case can be obtained with the help of the formula

$$-\frac{1}{\rho} \frac{dE}{dx} = \frac{N}{A} m_0 c^2 \int_0^{\epsilon_0} h \frac{d\sigma}{dk} dk. \quad (2.10)$$

The integral, which stands in right side (2.10), according to (1.35), coincides $(\epsilon_0 + 1) \Phi$, where Φ — the total cross section of bremsstrahlung. Therefore average/mean losses can be easily calculated, if are known formulas for Φ .

For $\epsilon \gg 1$ they use following formula

$$-\frac{1}{\rho} \frac{dE}{dx} = \frac{4Nr_0^2}{137A} m_0 c^2 Z^2 (\epsilon_0 + 1) \ln \left[2(\epsilon_0 + 1) - \frac{1}{3} \right], \quad (2.11)$$

which is obtained during use of formula (1.37). We give in Tables 3 and 4 applications/appendices of the value of ionizing losses

$\left(-\frac{1}{\rho} \frac{dE}{dx}\right)_u$ and total losses $\left(-\frac{1}{\rho} \frac{dE}{dx}\right)_T$, designed with the help of formulas (2.7) and (2.11). These values can be used for the rapid rough estimates of the losses of energy ΔE in the small route segments Δx .

$$\Delta E = \left(-\frac{1}{\rho} \frac{dE}{dx}\right) \Delta x. \quad (2.12)$$

Frequently results of experiment or calculations according to passage of electrons through substance it is possible to describe by universal by curves or formulas, if distance, passed of electrons, is expressed in units of complete landing run. By complete landing run is understood the maximum distance, passed by electron, if it moved along straight line and it lost energy continuously.

Complete landing run in g/cm² is computed from formula

$$R_0(E_0) = \int_0^{E_0} \left(-\frac{1}{\rho} \frac{dE}{dx}\right)_n^{-1} dE, \quad (2.13)$$

where

$$\left(-\frac{1}{\rho} \frac{dE}{dx}\right)_n = \left(-\frac{1}{\rho} \frac{dE}{dx}\right)_u + \left(-\frac{1}{\rho} \frac{dE}{dx}\right)_T.$$

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The calculations of the complete landing runs in the wide region of energies from 10 keV to 1000 MeV for different substances are carried out by Berger and Seltzer [37]. Since these calculations are published in the special report, we give in Table 5 of the Appendix values of landing runs for several elements in the region of energies of electrons 0.2-10 MeV. For determination $R_0(E_0, Z)$ ($Z > 6$) the tabulated data can be interpolated.

§ 2. Fluctuations in energy losses.

In previous paragraph we considered average/mean energy losses of electrons. In actuality electron loses energy discretely, and are possible losses both large and it is smaller than the average. For the first time a question about the fluctuations in energy losses to the ionization and the excitation was examined by Landau [38].

Function of energy-loss distribution in transit through thin layer of substance is obtained by Landau under the assumption that a) losses to bremsstrahlung are small, b) probable loss of energy ΔE_e satisfies inequalities $\Delta E_e \ll E_0$ and $\Delta E_e \ll E_{ev}$ (E_{ev} — electron-binding energy in atom, $E_{ev} \approx I$).

In Landau's theory is not considered correction for effect of density and that it is more substantially, is not taken into consideration effect of multiple scattering of electrons in this layer. According to Yang [39], the account of the latter actually is reduced to the replacement of thickness of the layer Δt to efficient thickness $\Delta t_{eff}(\Delta t, \phi_{ph}, \dots, \Delta t)$.

By most correct method account of multiple scattering can be carried out with the help of calculation by Monte Carlo.

Attractiveness of Landau's method consists in the fact that

probability of energy loss in the range $\Delta E - \Delta E + d(\Delta E)$ succeeds in expressing through universal function $w_L(\lambda)$, where λ - dimensionless parameter.

$$w(\Delta E) d(\Delta E) = w_L(\lambda) d\lambda, \quad (2.14)$$

and

$$\lambda = \frac{\Delta E - ax \left[\ln \frac{2m_0 c^2 \beta^2 ax}{(1-\beta^2) I^2} - \beta^2 + 0,423 \right]}{ax}, \quad (2.15)$$

where $a = 0.153 Z/A\beta^2$ (MeV·cm²/g), x - thickness of the layer in g/cm². The detailed tables of the values of function $w_L(\lambda)$ are given in the work of Böhrsh-Supan [40].

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For value $\lambda = -0.05$ function $w_L(\lambda)$ has maximum. Energy losses, which correspond to the maximum of distribution, is conventionally designated as the probable energy losses. From (2.15) with $\lambda = -0.05$ we obtain formula for the probable losses

$$\Delta E_p = ax \left[\ln \frac{2m_0 c^2 \beta^2 ax}{(1-\beta^2) I^2} - \beta^2 + 0,373 \right]. \quad (2.16)$$

Knowing the function of the distribution of losses, it is easy to find the average/mean energy loss in the form

$$\overline{\Delta E} = ax \left[\ln \frac{2m_0 c^2 \beta^2 E_0}{(1-\beta^2) I^2} - \beta^2 \right]. \quad (2.17)$$

It is important to note that the probable energy loss does not coincide with the average.

From asymmetry of curve $w_L(\lambda)$ it follows that for small x average/mean losses are more than probable, and this is connected with noticeable probability of large losses of energy $\Delta E > \Delta E_p$. However, $\frac{d}{dx} \overline{\Delta E} = \frac{d}{dx} \Delta E_p$.

with an increase in the thickness of absorber ΔE , ~~it~~ increases more rapidly than $\overline{\Delta E}$ due to the further logarithmic dependence on x and with the the large $x(ax > E/2)$ distribution curve becomes close to the Gaussian, while $\Delta E \approx \Delta E_0$.

In calculations by Monte Carlo it is necessary to know function of energy-loss distribution, greater than given, i.e., integral function. The probability of energy loss, which exceed ΔE , is determined with the help of the integral

$$\psi(\Delta E) d(\Delta E) = d(\Delta E) \int_{\Delta E}^{\Delta E_{\max}} w(\Delta E') d(\Delta E'). \quad (2.18)$$

Landau's theory was improved in works of Vavilov [41], Symon [42], Blunck and Leisegang [43]. The distributions of losses according to Vavilov are tabulated in the report of Seltzer and Berger [44] and more frequently they are used for the heavy charged/loaded particles. For Symon it was possible to consider the case, when energy losses were compared with the wave energy of electron, i.e., when the thickness of absorber no longer was small. The distribution function in this case is represented by the set of curves with different degree of the asymmetry: from the gaussian form with the large thickness to the distribution of Landau with the small thickness. Very work of Symon is published in the form of theses; therefore it is virtually unavailable, but the results of its calculations in the form of graphs and explanations to them are given in Rossi's book [45].

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Energy-loss distribution on Symon is given as the function of dimensionless parameter $\xi = \frac{\Delta E - \Delta E_0}{\Delta_0}$, where ΔE - energy losses, and ΔE_0 - probable losses, computed from the formula

$$\Delta E_0 = ax \left[\ln \frac{2m_e c^2 ax}{(1 - \beta^2)^{1/2}} - \beta^2 - j \right] \quad (2.19)$$

and

$$\Delta_0 = axb.$$

Family of graphs $w_e(\xi)$ is shown in Figure 9a, while family of integral function $\psi_e(\xi)$ - in Figure 9b. In order to use these graphs, it is necessary to at first compute value $G = \frac{ax}{E_m}$, where E_m - maximum energy, transmitted to the electron (it is computed from formula 1.16).

It is further necessary to find parameters j , b , λ , depicted on graphs of Figure 10 as functions G . In conclusion is computed value ξ , and from it and $w_e(\xi)$ (in Fig. 9a in right-hand corner is shown normalizing factor F , which is function λ). For convenience in the use of the distribution of Symon we give 6 applications/appendices of the value of integral function $\psi_e\left(\frac{\Delta E - \Delta E_0}{\Delta_0}\right)$ in the Table.

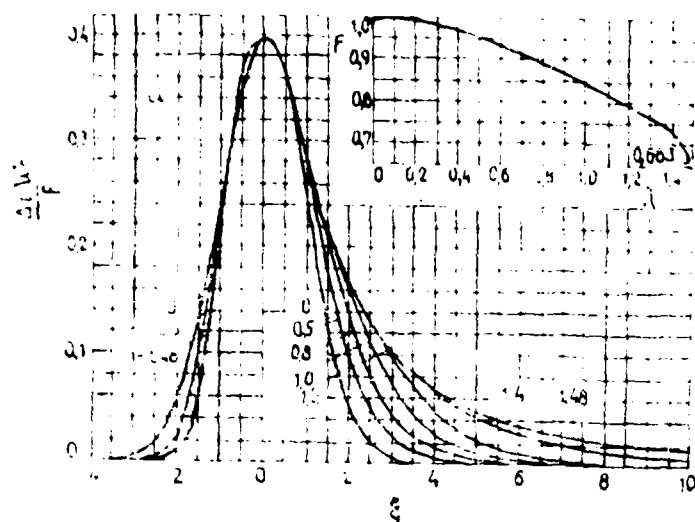


Fig. 9a. Graphs of the energy-loss distributions $w(\xi)$ on Symon. Numerals in the figures - value of the parameter λ . - normalizing factor.

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Blunck and Leisegang taught resonance interaction of falling/incident electron with inner electrons of atom. The function of the energy-loss distribution in this case is obtained somewhat wider than the corresponding functions of Landau or Symon. The distribution of Blunck-Leisegang is family of curves, depending on parameter b^2

$$b^2 = q\Delta E Z^2 (ax)^{-2}, \quad (2.20)$$

where $q \approx 20$ eV.

Very distribution function is sum of four gaussian curves and is written/recorded in the form

$$w(\Delta E) d(\Delta E) = \sum \frac{c_i t_i}{(t_i^2 + b^2)^{1/2}} \exp \left[-\frac{(t - t_i)^2}{t_i^2 + b^2} \right] dt, \quad (2.21)$$

where λ - dimensionless parameter, determined by expression

$$\lambda = \frac{\Delta E}{ax} \left[\ln \frac{E_0}{ax} - 1.116 \right], \quad (2.22)$$

and constants c, t, λ are given in Table 6.

Let us note that cases of large energy losses in distribution (2.21) are not examined.

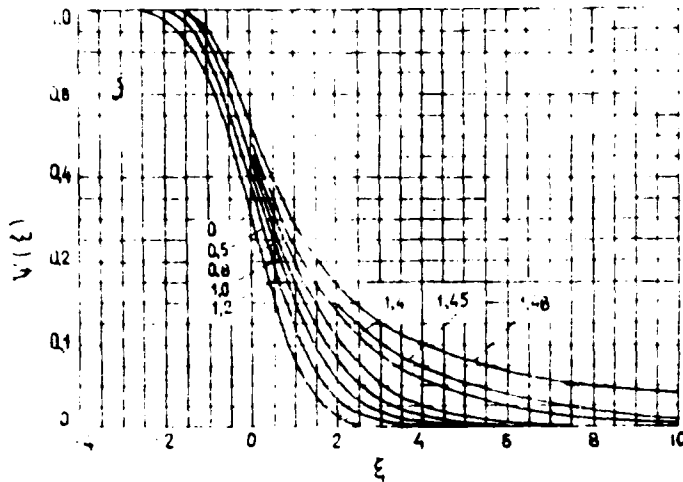


Fig. 9b. Graphs of integral distributions $\Phi(\xi)$ on Symon.

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When $b^2 \ll 3$, theory of Blunck and Leisegang brings to coinciding with theory Landau to distribution. The role of resonance broadening becomes noticeable only with the the large b^2 , i.e., for the relatively small thicknesses.

By deficiency in distribution in question in calculations Monte Carlo is use of considerable space of working storage of computer(s) for storing family of curves.

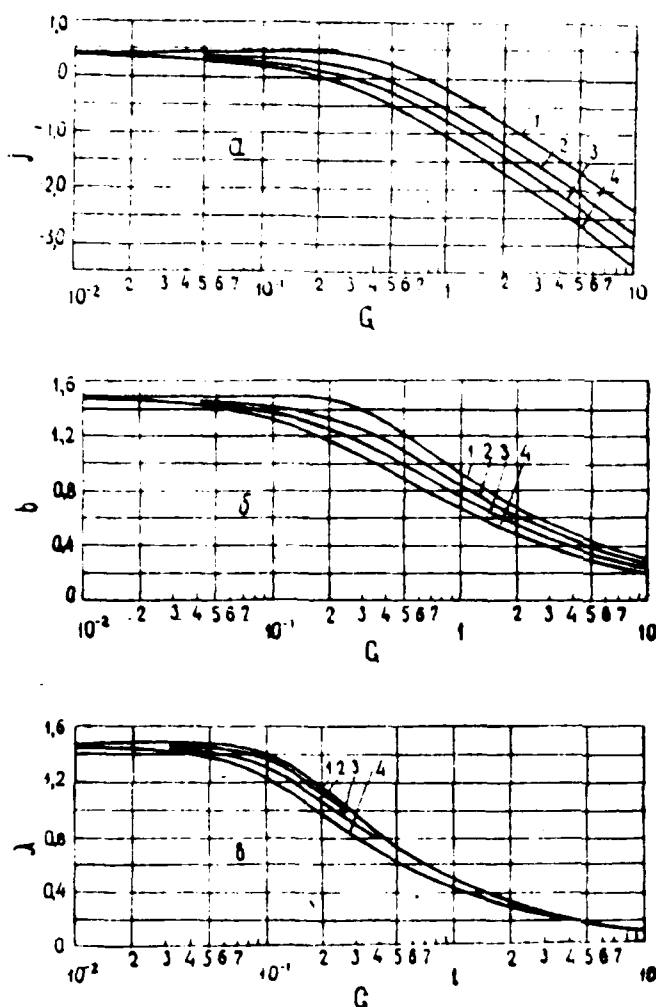


Fig. 10. Graph/diagrams of dependence: a) $j(G)$, b) $b(G)$, c) $\lambda(G)$. Curves 1, 2, 3, 4 correspond to values $\beta^2=0; 0.4; 0.7; 1$.

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Early experiments [46] agreed well with Landau's theory, if we in latter consider "effect of density" and to take into consideration multiple scattering and bremsstrahlung. In later works [47, 48] it was noted that during the use of instruments with the best energy permission/resolution for the agreement of experiment with the theory

it is necessary to calculate the resonance broadening of distributions on the basis of Blunck and Leisegang.

Energy-loss distribution for positrons was for the first time examined in work Rohrlich and Carlson [49]. Landau's method was used, and for the scattering cross section of positron on the electron was used formula (1.26) of Bhabha. For the probable energy loss in this work the expression

$$\Delta E_p^* = ax \left[\ln \frac{2m_0 c^2 \beta^2 ax}{(1-\beta^2) I^2} - \beta^2 + 0,373 - 2,8\delta \right],$$

where

$$\delta = \frac{2\pi e^4 n_0 Zx}{i(m_0 c^2)^2} \left[2 - \frac{1}{(\epsilon+2)^2} \right]. \quad (2.23)$$

is obtained

Nishi and Hiromasa [50] carried out experimental check energy-loss distributions of positrons with energy 0.3 MeV in different substances, indicate that theory [49] is valid only for small thicknesses and that with increase in thickness experimental distribution very well will be coordinated with distribution (2.21), if average/mean energy losses are computed on (2.7) with $f^*(\epsilon)$. Thus, distribution (2.21), which more fully considers all effects, connected with the fluctuations of energy loss, can be used also for the positrons. In the calculations by Monte Carlo, where, as a rule are examined thin layers, for the positrons will be valid and the theory of Symon.

To fluctuations are subjected other losses of energy - radiation losses. The function of the distribution of these losses they for the first time computed Bethe and Heitler [25].

Table 6.

	1	2	3	4
c_1	0,174	0,068	0,019	0,007
λ_1	0,000	8,000	6,500	11,000
λ_2	1,800	2,000	3,000	5,000

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They proceeded from simplified formula (1.31) for the differential cross section of bremsstrahlung. Expression for the probability of the losses of energy ΔE_T takes the form

$$\omega(E_0, \Delta E, t) d(\Delta E) = \frac{d(\Delta E)}{E_0} \frac{\left[\ln \frac{E_0}{E_0 - \Delta E} \right]^{\alpha x - 1}}{\Gamma(\alpha x)}, \quad (2.24)$$

where $\alpha = 1,4 \cdot 10^{-3} \frac{Z^2}{A} \left[\frac{4}{3} \ln 183 Z^{-1/2} + \frac{1}{9} \right] \left(\frac{cM^2}{e} \right)$; $\Gamma(\alpha x)$ -- gamma function.

Distribution $\omega(E_0, \Delta E, t)$ is calibrated to one. The form of the function is such, that when $\alpha x > 1$, more probable are the large losses as a result of a few reports/events of interaction, and when $\alpha x \ll 1$, more probable become the low losses of energy. This latter/last case most frequently is realized in the calculations by Monte Carlo. Until now, we examined both sources energy losses independently.

It is more right, as this is shown by Blunck and Westphal [51], to seek function of distribution of total losses due to both processes. It is necessary, consequently, to find the probability of the given losses ΔE_{Σ} knowing the functions of the distribution of losses to the bremsstrahlung and to the ionization. The unknown distribution is located with the help of the convolution integral in this form:

$$w(\Delta E_n) = \int_0^{\Delta E_n} w(\Delta E_n - \gamma) \omega(\gamma) d\gamma, \quad (2.25)$$

where

$$\Delta E_n = \Delta E_n - \gamma.$$

In work [51] families of these distributions are designed, using as $w(\Delta E_n - \gamma)$ distribution (2.21) and $\omega(\gamma)$ in the form (2.24). We led analogous calculations, but the energy-loss distribution to the ionization was undertaken on Symon. The graphs of the integral distributions

$$\psi(\Delta E_n) = \int_0^{E_0} w(\Delta E_n) d(\Delta E_n) \quad (2.26)$$

are shown in Figure 11.

On figure it is evident that for Pb($E_0=5.75$ MeV) role of fluctuations in radiation losses becomes essential (for comparison in the same figure shown distribution, which considers only fluctuations in ionizing losses). For Si($E_0=0.95$ MeV) distribution (2.26) coincides with the distribution, designed only for the ionizing losses.

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Let us note that even in the case of small thicknesses and relatively low energies $E \approx 2$ MeV average/mean energy losses, calculated for $Z > 50$ during use of distribution (2.24), are more than sum of average/mean energy losses, calculated separately from each other, which indicates importance of account of fluctuation of total losses.

Checking distribution (2.25) showed that it somewhat wider than the experimental [52, 53]. This is connected with the approximate character of expression for $\omega(\eta)$. It is probable that distribution of Symon in the combination with $\omega(\eta)$ will give better agreement with the experiment, since it is somewhat narrower than the distribution of Blunck and Leisegang. However, the further experimental check of this assumption is required.

§ 3. Multiple scattering of electrons.

Results, obtained in calculations by Monte Carlo, are to a considerable extent connected with detailed and checked during experiment theories of multiple scattering. We will consider here only two distributions for the angles in multiple scattering: Goudsmit-Saunderson and Moliere.

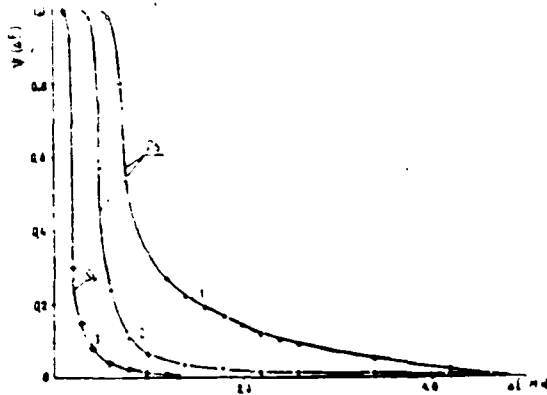


Fig. 11. Graphs of integral function of distribution $q(E)$ of energy loss of electron. 1 - total losses; 2 - loss only to the ionization ($E_0=5.75$ MeV in Pb). For Si and $E_0=0.95$ MeV both distributions coincide (3).

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1. Goudsmit and Saunderson [7] showed that the quantity of electrons scattered in interval of angles $\theta - \theta + d\theta$ after the passage of path segment t , is given by formula

$$f_{r-c}(t, \theta) \sin \theta d\theta = \sin \theta d\theta \sum_{l=0}^{\infty} (l + \frac{1}{2}) P_l(\cos \theta) \times \exp \left[- \int_0^t G_l(t') dt' \right], \quad (2.27)$$

where

$$G_l(t) = n_0 \int_0^{\pi} \frac{d\sigma(\theta, t)}{d\Omega} [1 - P_l(\cos \theta)] d\Omega. \quad (2.28)$$

Here $(d\sigma(\theta, t)/d\Omega)$ - section of single scattering, $P_l(\cos \theta)$ - Legendre's polynomial. The decrease of energy of particle due to the ionization losses and bremsstrahlung in distribution (2.27) is considered through the implicit functional dependence $t=t(E)$, moreover the continuous losses of energy are assumed. In such a manner both in the distribution for energy losses does not succeed in taking into account multiple scattering and in the distribution for the angles in repeated scattering it is impossible to correctly consider fluctuations in energy losses.

To advantages of distribution in question should be related, first, possibility of using any expressions for section of single scattering, secondly, applicability of it for any scattering angles, including large ($\sim 180^\circ$). Use in (2.27) of the Mott cross sections (1.2) makes it possible to consider the relativistic and spin effects,

essential for the large scattering angles, which lead to the difference in the passage of electrons and positrons through the substance into consideration.

Deficiency in theory of Goudsmit-Saunderson is weak convergence of series into (2.27) especially for small t and large θ , as a result of which for obtaining necessary accuracy it is necessary to add large number of terms of expansion (to 100). The latter so impeded the use of distribution that it in practice was not used in the calculations, on it served as the criterion of the validity of one or the other simplified variations of the theory of multiple scattering.

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Only after Spencer proposed the recursion relations, which make it possible to compute G_l with the high indices up to $l=100$, obtaining distribution (2.27) became relatively simple procedure.

Examination of Spencer's method [10] let us begin from calculation of integral

$$I = \int_0^t G_l(t') dt'. \quad (2.29)$$

Let us switch over to the new variable $s=(R_0-t)/(R_0)$, where R_0 is the total path of electron in the substance. Then

$$I = \int_0^t G_l(t') dt' = R_0 \int_s^1 G_l^*(s') ds'. \quad (2.30)$$

According to Spencer,

$$G_l^*(s) = G_l(s) \frac{G_l^*(1)}{G_l^*(1)}, \quad (2.31)$$

where

$$G_l^*(s) = \frac{C_1}{s(s+C_2)}, \quad (2.32)$$

and $G_l^*(1)$ and $G_{l+1}^*(1)$ coincide with the values of function G_l of the corresponding order, calculated for the wave energy of electron. Substituting (2.32) and (2.31) in (2.30), it is possible unknown integral (2.29) to register in the form

$$I = \frac{R_0 C_1}{C_2} \cdot \frac{G_l^*(1)}{G_{l+1}^*(1)} \ln \frac{s+C_2}{s(1+C_2)}. \quad (2.33)$$

Parameters C_1 and C_2 can find, solving system of equations with the the known $G_{l+1}^*(s)$ and $G_{l+1}^*(1)$:

$$\begin{aligned} G_{l+1}^*(s) &= \frac{C_1}{s(s+C_2)} \\ G_{l+1}^*(1) &= \frac{C_1}{1+C_1}. \end{aligned} \quad (2.34)$$

Let us consider the case, when $t/R_0 \ll 1$, i.e., $s \approx 1$, being usually realized in the Monte Carlo calculations. Then, after being bounded to the first term of expansion $\ln \frac{s+C_2}{s(1+C_2)}$ in the Maclaurin series, after simple transformations we obtain convenient approximation/approach for I

$$I = G_{l+1}^*(1) t - G_l(t=0) t. \quad (2.35)$$

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Let us find now expression for G_l , using as section of single scattering Mott's formula (1.11) with correction term, described by formula (1.12). After passing to new variable $u = \cos \theta$ and after

fulfilling in (2.28) the integration for the azimuthal angle, we will obtain

$$G_l = 4\pi e^2 Z(Z+1)(1-\gamma) \left\{ Q(-2, l) \int_{-1}^1 x \cos x \right. \\ \left. - h_1 \left[Q(-\frac{3}{2}, l) + h_2 Q(-1, l) - h_3 Q(-\frac{1}{2}, l) \right] \right. \\ \left. h_4 Q(0, l) - h_5 Q(\frac{1}{2}, l) \right\}. \quad (2.36)$$

The confronting in curly braces (2.36) polynomials $Q(m, l)$ are expressed as polynomials $(1-u+2\gamma)^m$ with the help of the relationship/ratio

$$Q(m, l) = \int_{-1}^1 (1-u+2\gamma)^m [1 - P_l(u)] du, \quad (2.37)$$

where γ - parameter of shadowing.

On the basis of general/common properties of polynomials of Legendre, Spencer gives following recursion relations for $Q(m, l)$.

The first makes it possible to pass from m to $m+1$:

$$Q(m+1, l) = (1+2\gamma)Q(m, l) + Q(m, 1) \\ - \frac{l+1}{2l+1} Q(m, l+1) - \frac{l}{2l+1} Q(m, l-1). \quad (2.38)$$

moreover $Q(m, 0) \equiv 0$.

The second makes it possible to pass from l to $l+1$ (for this m), moreover, if m - whole, then is used (2.39), and if m - fractional - (2.40):

$$lQ(-2, l+1) = (2l+1)(1+2\gamma)Q(-2, l) - (l+1)Q(-2, \\ l-1) - (2l+1)(1+\gamma)^{-1}, \quad (2.39)$$

$$Q\left(-\frac{3}{2}, 1\right) = \frac{1}{2}Q\left(-\frac{3}{2}, 1\right) + Q\left(-\frac{3}{2}, 1\right). \quad (2.40)$$

where $l = 1$, a $\tilde{\eta} = 1 - 2\eta\left(\sqrt{1 + \frac{1}{\eta}} - 1\right)$.

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The initial values $Q(-2.1)$ and $Q(-3/2), 1)$ necessary for using the formulae above are computed from the formulas

$$Q(-2, 1) = \ln\left(1 + \frac{1}{\eta}\right) -$$

$$-(1 + \eta)^{-1},$$

$$Q\left(-\frac{3}{2}, 1\right) = 2(2\tilde{\eta})^{-1}(1 +$$

$$+ \tilde{\eta})^{-1}. \quad (2.41)$$

We already indicated weak convergence of series. Therefore for obtaining the necessary accuracy, especially in the region of large angles θ , it is necessary to count to 100 members of series/row (2.27).

Table 7 shows convergence of values $f(\theta, t)$, calculation is carried out for Al, ~~by~~ $E_0 = 1$ MeV, $t = 0.03$ g/cm².

Moliere's theory [8] is simpler theory of multiple scattering, strictly speaking, applied for small scattering angles ($\theta < 1$ rad). The latter made it possible to create the completed theory and to obtain the distribution function for the scattering angles for the scattering

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angles in the analytical form.

Table 7. Convergence of series for $f_{\pi}(0)$ as the function of a number of terms l .

l	$n=18$	45	72	90	135	171
10	3.2723697	0.52174434	-0.46401429	0.39853767	-0.23270203	-0.10793879
20	3.6323317	0.17457685	-0.06470117	-0.05865163	0.03853396	0.01550726
30	3.4721717	0.06237644	0.00305118	0.01081927	0.00403899	0.00173254
40	3.4669253	0.06750488	0.01025154	0.00349300	-0.0000652	-0.00016018
50	3.4684676	0.06871539	0.01097318	0.00424696	0.00028294	-0.00000999
60	3.4687265	0.06865563	0.01105342	0.00415893	0.00033319	0.00001282
70	3.4686945	0.06863593	0.01106404	0.00417140	0.00032769	0.00001057
80	3.4686932	0.06863721	0.01106576	0.00416921	0.00032640	0.00000999
90	3.4686946	0.06863796	0.01106610	0.00416969	0.00032652	0.00001006
100	3.4686946	0.06863783	0.01106618	0.00416956	0.00032665	0.00001010

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Thoroughly analyzing theory of Moliere, Bethe [54] showed that it is possible to introduce corrective factor into distribution function, making it possible to spread theory to large scattering angles and thus to make its universal just as theory of Goudsmit-Saunderson. Below we still will return to the comparison of results of both theories, and now we will proceed to the examination of the fundamentals of Moliere's theory.

The presence of only one parameter, so-called angle of shadowing (1.7) is most important result of theory, which makes it attractive for experimenters. Further, the distribution function does not depend on the form of the section of single scattering; only requirement for the section so that it would pass at the large scattering angles in the section of Rutherford (1.1). Let us note immediately that this assumption, as it follows from the discussion, carried out in Chapter

I, in reality occurs. For the probability of electron scatterings in interval of $\theta - \theta + d\theta$ after the passage of layer t Moliere gives the expression

$$f_M(\theta, t) d\theta = [f^{(0)}(\vartheta) + B^{-1} f^{(1)}(\vartheta) + B^{-2} f^{(2)}(\vartheta) + \dots] d\vartheta. \quad (2.42)$$

Condition for applicability (2.42) to small angles is satisfied by replacement of $\sin\theta \approx \theta$. For obtaining the accuracy of order 1% it suffices to consider in (2.42) only three first terms.

$F^0(\vartheta)$ - Gaussian type function, and functions $f^1(\vartheta)$ and $f^2(\vartheta)$ consider scattering of large angles and divergence from Born approximation. The numerical values of these functions, refined by Bethe, are given in the Table of 7 applications/appendices. dimensionless variable ϑ is connected with the scattering angle θ and the parameter of theory B with the relationship/ratio

$$\vartheta = \frac{\theta}{\sqrt{1 + \frac{1}{B^2}}}, \quad (2.43)$$

where

$$\frac{1}{B^2} = \frac{4\pi n_0 t e^4 Z(Z+1)}{p^2 c^2 \lambda^2}. \quad (2.44)$$

In formula (2.44) and in those following it, the correction for mentioned in Chapter I inelastic scattering of electrons $Z^2 \rightarrow Z(Z+1)$ is taken into consideration. The new parameter B is obtained from the solution of the transcendental equation

$$B - \ln B = b, \quad (2.45)$$

where

$$b = \ln \frac{\lambda_e^4}{1.167 \lambda_0^4} = \ln \frac{6880 (Z+1) Z^2}{3^2 A (1+3.34 Z^2)} \quad (2.46)$$

it is expressed as the angle of shadowing $\lambda_0^2 = 4\gamma$, and γ is described by formula (1.7). Values B usually lie/rest at interval of 5-20.

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Let us note that value $\Omega = e^b$ gives exemplary/approximate collision frequency, experienced/tested by electron with passage of thickness of the layer t . Since by its nature Moliere's theory statistical, then collision frequency must be large, is usually more than 20. This is one of the limitations of theory. On the other hand, collision frequency and, consequently, the thickness of the layer cannot be large so that the distribution would not be distorted due to the disregard of energy loss to the ionization.

Bethe in work [54] showed that if we use in both theories of Moliere and Goudsmit-Sauderson the Rutherford scattering cross section, then between appropriate distributions there is dependence

$$f_{RC}(\theta, t) = \left(\frac{\theta}{\sin \theta} \right)^{1/2} \exp \left[-\frac{1}{16} \chi_e^2 B \right] f_M(\theta, t) \cdot \frac{1}{24}. \quad (2.47)$$

This formula is valid as long as $\chi_e^2 B \ll 1$, thus far the width of the gaussian part of the distribution of Moliere ≤ 1 rad. But in this case $\left[\frac{1}{16} \chi_e^2 B \right] \approx 1$. Without the large error it is possible to disregard in (2.47) and $1/24$. Then the improved distribution function, spread to the region of the angles of $\theta > 1$ rad, is located with the help of the formula

$$f_E(\theta) \sin \theta d\theta = \theta d\theta \left[\frac{\sin(\gamma_c \sqrt{B} \theta)}{\gamma_c \sqrt{B} \theta} \right]^2 \cdot [f^{(0)} + B^{-1} f^{(1)} + B^{-2} f^{(2)}] \quad (2.48)$$

Obviously, correction factor $\left[\frac{\sin(\gamma_c \sqrt{B} \theta)}{\gamma_c \sqrt{B} \theta} \right]^2$ although improves distribution of Moliere, it cannot with full weight contain behavior of function for large scattering angles. Is more than that, scattering cross section to the large angles is sensitive to the shadowing, and the atom model according to Thomas-Fermi, which is used in Moliere's theory, it does not transmit this dependence. In Figure 12 we are congruent/equate distributions (2.48) and (2.27). The latter was calculated, using a section of Mott.

From figure it is evident that in region of large angles for Au difference between both distributions to become essential. One should indicate one more deficiency in Moliere's theory - in it the difference in the angular distributions for the electrons and the positrons is not exhibited.

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Development of Moliere's theory is given in works of Nigam et al. [9]. First of all it seemed that the shielded scattering cross section, obtained by Moliere during the use for the atom of Thomas-Fermi model, is imprecise in all orders $\alpha = Z/137$, except zero. Correct expression for the section can be obtained by the method, proposed by [55].

Nigam et al. use relativistic scattering cross section of Dalitz [56] with shielded Coulomb potential. Although this potential is worse than Thomas-Fermi potential, the procedure of calculation of the distribution function according to Nigam is such, which ensures coincidence with a precise distribution of Goudsmit-Saunderson.

Angle of shadowing according to Nigam $\gamma_N^2 = 4\gamma_M$, where γ_M is given by formula (1.8). The difference between the angles of shadowing according to Moliere and Nigam sufficiently substantially for large Z and low energies is shown in Table 1. Since into the formula for the angle of shadowing (1.8) α enters to the first degree (α^2 - into the theory of Moliere), in the distribution function is exhibited the difference between the electrons and positrons [57], not observed in Moliere's theory.

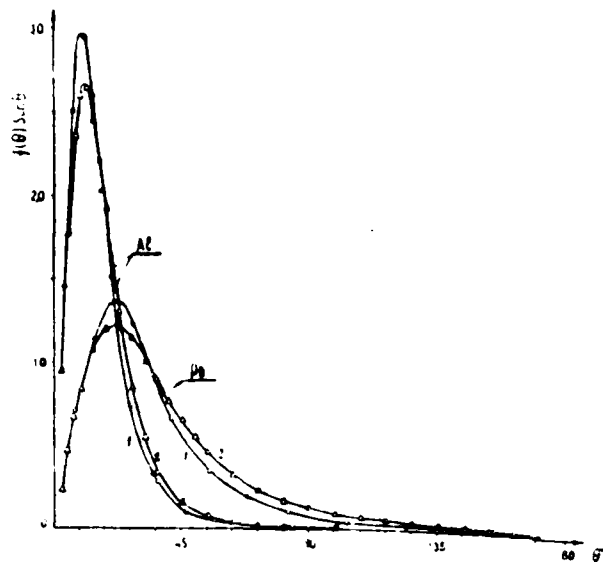


Fig. 12. Comparison of the function of distribution $f(\theta)\sin\theta$, calculated for Al and Pb $E_0=0.2$ MeV: 1 - according to Moliere; 2 - according to Goudsmit-Saunderson.

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Function of distribution was obtained in second Born approximation/approach can be represented in the form, analogous to distribution of Moliere

$$f(b, t) = \frac{K}{k_c^2 B} \left[f^{(0)} + \frac{1}{B} (f^{(1)'} + f^{(1)}) + \frac{1}{2B^2} (f^{(2)'} + f^{(2)}) \right], \quad (2.49)$$

where $f^{(0)}$, $f^{(1)}$ and $f^{(2)}$ - Moliere-Bethe functions (see Table 7 application/appendix); $f^{(1)'}$ and $f^{(2)'}$ - corrections, which appear due to second Born approximation

$$f^{(1)'} = \int_0^{\infty} \varphi(u) [-\pi \alpha^2 \lambda_c \sqrt{Bu}] du \quad (2.50)$$

and

$$f^{(2)} = \int_0^{\Gamma} \varphi(u) \left[-\frac{1}{2} \pi \beta \xi u^2 / \epsilon + B \ln \left(-\frac{1}{4} u^2 \right) \right] du, \quad (2.51)$$

where

$$\varphi(u) = \exp \left[1 + \frac{1}{2} \chi_c^2 \left(\beta^2 + \pi \alpha \beta - \frac{1}{4} \xi \right) \ln u \right] \cdot J_0 \left(\frac{\eta u}{\chi_c + B} \right) \exp \left(-\frac{1}{4} u^2 \right). \quad (2.52)$$

J_0 - Bessel function, and χ_c is found from formula (2.44). Other parameters, necessary for calculation $f^{(1)}$ and $f^{(2)}$, are determined by the expressions

$$\xi = 1 + 1,34 \cdot 10^{-5} \frac{Z^{1/2}}{\beta^3} (1 - \beta^2)^{1/2},$$

$$\Gamma = \frac{\chi_c + B}{\chi_0} + \frac{1}{2} \chi_c \sqrt{B}, \quad (2.53)$$

$$\chi_0 = 1,26 \cdot \frac{h}{2\pi} \frac{Z^{1/2}}{a_c P}.$$

For the coefficient K of Nigam it gives the expression

$$K = \exp \left\{ \frac{1}{16} B \chi_c^2 \left(1 + \frac{8\pi \alpha \beta}{B} + \frac{2\xi}{B} \ln 2 + \frac{8}{B} (\beta^2 + \pi \alpha \beta) [0,5772 - \ln(\chi_c + B)] \right) \right\}. \quad (2.54)$$

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Parameter B is found from the solution of the transcendental equation

$$B - \xi \ln B = b, \quad (2.55)$$

where

$$b = \xi \ln \left(\frac{1}{4} \chi_c^2 \right) - \ln \left(-\frac{1}{4} \chi_c^2 \right),$$

and χ_c is computed from the formula

$$\ln \left(\frac{2}{f_1} \right) = \ln \left(\frac{2}{f_1} \right) + 0.8456 \left(\frac{2}{f_1} \right) (1 - \beta^2) - 0.0772. \quad (2.56)$$

Dimensionless variable in theory of Nigam, just as in Moliere's theory, is value $\theta = \frac{q}{k c^2 B}$. Marion and Zimmerman [58] they considered the simplified variation of the theory of Nigam. Actually, in the region of energies 0.4-6.0 MeV examined/considered by us for all elements, including $Z=82$, $\xi \approx 1$, $k_0 \approx k$, $K \approx 1$, $k_c \approx 10^{-4}$. Then terms $f^{(1)}$ and $f^{(2)}$ can be disregarded/neglected respectively in comparison with $f^{(0)}$ and $f^{(3)}$, (2.49) it is possible to register in the form

$$f(\theta) = \frac{1}{k c^2 B} \left[f^{(0)} + \frac{1}{B} f^{(1)} + \frac{1}{2B^2} f^{(2)} \right], \quad (2.57)$$

moreover in this case in it is found from the equation

$$B - \ln B = b$$

with

$$b = \ln \left[\frac{2730 (Z+1) Z^{-1/2}}{A^{3/4}} \right] - 0.1544. \quad (2.58)$$

Table of 8 applications/appendices gives normalized functions $f(\theta)$ (they are calibrated to $f(0)$) for several values of B :

$$B = 4, 5, 6, 7, 8, 9, 10, 12.$$

Calculations [58] were carried out for protons, but their applicability to electrons was not limited by anything, since formula of Dalitz for scattering cross section was derived for particles with spin 1/2. The experimental check of the angular distributions of stray electrons confirms the theory of Nigam, although in the majority of cases is a completely satisfactory agreement also with Moliere's

theory. A good agreement of experiment is obtained also with the theory Gaudsmit- Saunderson. This indicates that the theories of multiple scattering examined at present can ensure the reliable data about the angular distributions, passing through the substance of electrons.

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And nevertheless we give preference in our calculations to the theory of Goudsmit-Saunderson, which does not require any approximations/approaches and gives a "precise" angular distribution, if a precise section of the elementary report/event of scattering is known.

Only deficiency in this theory, being more of a psychological nature, is need for numerical calculation of distribution. However, many problems, which were being earlier considered hopeless (in the sense of the analytical representation of the solution), are solved rapidly and efficiently with the help of the owls of belt computational methods.

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Chapter III.

ANALYTICAL AND NUMERICAL METHODS OF THE STUDY OF THE TRANSFER OF ELECTRONS IN A SUBSTANCE.

In this chapter range of applicability of analytical method of study of transfer of electrons, based on solution of kinetic equation, briefly is examined. Primary attention is paid to the method of Monte- Carlo: its special features in application to electrons are explained. Are examined the basic versions of the method, when tracking is conducted by a) by consecutive collisions; b) with the use of theories of multiple scattering. The latter/last version and its modifications, used by different authors, in detail is analyzed. The methods of accelerating the calculation are considered.

§ 1. Kinetic equation of transfer.

Kinetic equation for electrons has the same structure, as equation for γ -quanta, and it is written/recorded in general case as follows:

$$\frac{1}{v} \frac{\partial F}{\partial t} + \vec{u} \nabla F + \mu F = \int_E^{\infty} dE' \int_{4\pi} du' > \\ < F(E', u', r', t) \phi(E', u' \rightarrow E, u). \quad (3. 1)$$

Here v - velocity of electron, $F(E, \vec{u}, \vec{r}, \vec{t}) dE_{\lambda} d\vec{u}$ - particle flux at the moment of time t at point \vec{r} of the spaces, energy and angle of which lie/rest respectively at intervals of $E-E+dE$ and $\vec{u}-\vec{u}+d\vec{u}$, $\psi(E, \vec{u}' \rightarrow E, \vec{u})$ - the probability (per unit of path) of the fact that the particle, which has energy and angle E', \vec{u}' , will change them as a result of collision for E, \vec{u} .

FOOTNOTE ¹. Here by \vec{u} is understood the pair of angles θ and ϕ in the spherical coordinates. ENDFOOTNOTE.

Usually temporary/time term $\frac{1}{v} \frac{\partial F}{\partial t}$ is substituted by the term $\partial F / \partial R$, where R is the path of particle with this energy (speed) $R = \int_0^t v(t') dt'$.

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Introduction of a new variable allows us to pass by variable R in (3.1) to so-called approximation/approach of continuous losses of energy, when energy and path are connected by a unique analytical dependence. Then from the complex dependence for $\psi(E', \vec{u}', \rightarrow E, \vec{u})$ it is possible to isolate the energy part (it is expressed as function R), and only angular dependence remains under the integral.

This approximation/approach, for the first time proposed by Lewis [59], applicably only to infinite medium and in it are not considered fluctuations in energy losses to ionization, or secondary radiation (δ - electrons, bremsstrahlung). The solution of the equation of transfer are expressions for the spatial moments of the unknown

distribution (spectral, absorbed energy, etc.), and sequence of an infinite number of moments/torques, as is known, it is equivalent to the unknown distribution. In actuality it is possible to calculate only the finite number (usually not more than 10) of moments/torques, and, consequently, the distribution constructed with their aid will be incorrect, especially at the large depths in the substance.

Spencer [10] showed that knowledge of asymptotic properties of moments/torques and some special features of unknown distribution at large depths makes it possible to use methodology, worked out for γ -quanta [60], and to calculate function of distribution of absorbed energy ².

FOOTNOTE ². In the principle it is possible to obtain the spectrum of electrons at the assigned depth, but due to the utilized approximations/approaches hardly it is possible to expect even qualitative agreement with the experiment. ENDFOOTNOTE.

The distribution obtained by Spencer for the point and plane sources of electrons is in a good agreement with the experiment everywhere, besides the boundaries of layer. After the publication of the work of Spencer it was very little done for further development of the methods of solving equation (3.1).

Among scarce works, dedicated to this question, should be noted works of Adawi [61], where is done attempt to take into account

bremsstrahlung, and Crew [62], which solved two-dimensional problem and were obtained isodose fields in light media.

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The simplified model of a deep penetration of electrons was developed by Yevdokimov [63]. At the base of its model lies/rests the diagram, very close to that utilized in the Monte Carlo method, according to which electron path is broken line, which consists of the rectilinear segments, the direction of each of which is located by the averaging of angular distribution (usually in the approximation/approach of small angles). For this model it is possible to register the kinetic equation, whose solution can be obtained only at the large depths.

Thus, at present there does not exist detailed mathematical apparatus for solution of equation (3.1). Taking into account specific difficulties connected with the role of the boundaries (for the charged/loaded particles, which have small path, this is substantial), it is hardly possible to expect that the future theory will be able to compete with the Monte Carlo method, which gives the sufficiently exact solution of any problems, although without the refinement, inherent in a precise theory. In this case the goal justifies means.

§ 2. Monte Carlo method in the problems of the transfer of electrons.

It is difficult to give any precision determination of Monte

Carlo method. This is caused by the variety of problems decided with its aid. However, sufficiently fully its essence can be determined as follows: the Monte Carlo method is the numerical method of the solution with the help of the random tests of certain circle of the physical and mathematical problems, which carry the probabilistic (stochastic) character, or by which it is possible to compare probabilistic analog. In detail the proof of method and the field of its applicability is stated in books [64-66] and group of the works, published in collectors/collections [67, 68].

This above determination assumes two forms of problems, solved by Monte Carlo method. First of all this of the problem of radiation transfer and number of the problems of nuclear physics, which are in the essence probabilistic. In this case their solution is reduced to the selection from the appropriate random number distributions so, that their mathematical expectation coincides with the unknown value a . The mathematical expectation $M[\xi]$ is considered as arithmetic mean of the selections of the random variables: $M[\xi] = \frac{1}{n} \sum_{i=1}^n \xi_i$.

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On the basis of the central limit theorem it is possible to claim that the distribution arithmetic mean values asymptotically tends for the normal distribution and $M[\xi] \rightarrow a$ (on the probability). Error in this case does not exceed value $\beta[D[\xi]/N]^{1/2}$, where β - constant, determined by authenticity level of evaluation/estimate, but $D[\xi]$ - dispersion of random variable ξ [69].

$$D[\xi] = M[\xi^2] - M^2[\xi]. \quad (3.2)$$

Another circle of problems does not have straight/direct probabilistic character as, for example, resolution of Laplace's equation in electromagnetic field theory. For this equation there does not exist the analytical solution under the assigned boundary conditions. On the other hand, the same equation of Laplace describes the diffusion of particles in the region, limited by the absorbing barriers. Therefore to Laplace's equation in the electromagnetic theory it is possible to compare as the analog the equation for the diffusion of particles, which already can be solved by the Monte Carlo method. Many problems of gas- and of hydrodynamics, applied mathematics are solved thus such.

Let us return to our problem of transfer of electrons in substance and will consider simple problem. Suppose that on a plate of the specific thickness which is infinite in two measurements, a charged particle beam (electrons, protons) with known energy spectrum and the angular distribution falls. It is necessary to determine a number of particles, which passed through the plate, their energy and angular distribution.

Interaction of charged/loaded particles with substance is complicated stochastic process. Thus, electron can experience elastic scattering on the kernel, emit the quantum of bremsstrahlung, lose energy as a result of inelastic interaction with atom shell, and

proton, besides elastic and inelastic scatterings, can initiate the nuclear reactions (p, n) , $(p, 2n)$..., etc.

Each of enumerated reports/events of interaction is described by specific probability laws, obtained from theory or experiment. Other words, for each report/event are noted for only probability scatterings to the specific angle, the probability of this energy loss, the probability of the participation of particle in the specific nuclear reaction. During the solution of our problem by the Monte Carlo method complicated stochastic process is considered as the sequence of elementary actor (simple Markov chain), moreover the concrete/specific values of the parameters of the particles of such, as landing run, energy, direction of motion after scattering are obtained from the appropriate distributions with the help of the random numbers. The procedure of the selection of values from the probabilistic distributions with the help of the random numbers, according to the conventional terminology, is called drawing.

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Thus, step by step with the help of drawings is realized complicated stochastic process. The overall diagram of calculation by Monte Carlo in our case must include the following steps: 1) the drawing of wave energy and angle of incidence from energy-angular source distribution; 2) the drawing of landing run before first interaction from the range distribution; 3) the drawing of the form of process (elastic or inelastic scattering, bremsstrahlung, nuclear

reaction, etc.). As a result of the previous drawing let it be established that the elastic scattering occurred, then from the appropriate distribution we generate energy of particle and compute scattering angle (it is possible to draw angle and to compute energy). Then procedure is repeated from point/item 2 for the particle, in which instead of the initial values of energy and direction of motion are used the drawn energy and angle as long as: a) energy of particle will not become less than the certain minimum value E_{min} , with which it is possible to consider it absorbed in the plate; b) particle will not leave the plate. In the latter case we simultaneously fix/record energy and direction of propagation. But if in the process of inelastic interaction, for example, nuclear interaction, appeared one or several secondary particles, then they continue from the collision to collision analogously with initial particle. Thus, the solution of problem with the help of the Monte Carlo method is reduced to the generation of the sequence of random variables $E_i, \vec{r}_i, \vec{u}_i$ ($E_n^{(i)}, \vec{r}_n^{(i)}, \vec{u}_n^{(i)}$), which characterize the so-called trajectory (history) of particle.

$$\begin{array}{l} E_0, E_1, \boxed{E_2}, \dots, E_n, \dots \\ \vec{r}_0, \vec{r}_1, \boxed{\vec{r}_2}, \dots, \vec{r}_n, \dots \\ \vec{u}_0, \vec{u}_1, \boxed{\vec{u}_2}, \dots, \vec{u}_n, \dots \end{array}$$

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$$E_0^{(i)}, E_1^{(i)}$$

$$\vec{r}_0^{(i)}, \vec{r}_1^{(i)}$$

$$\vec{u}_0^{(i)}, \vec{u}_1^{(i)},$$

where E - energy of particle;

\vec{r} - set of coordinates;

\vec{u} - pair of angles in the spherical coordinates.

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For explaining general laws governing passage of particle through plate it is necessary to realize large number of such trajectories and to subject to their statistical processing. Let us note that the results, obtained during this processing, contain all fluctuations, which associate real process.

By fundamental point in calculation procedure Monte Carlo is drawing of values of random parameters of particle $E_i, \vec{r}_i, \vec{u}_i$ from appropriate probabilistic distributions. In our case - these are angular distribution in multiple scattering and the energy-loss distribution in the inelastic scattering. Since the drawing is repeated repeatedly, its procedure must be sufficient efficient, i.e., on it must be spent little time. Let us consider one of the frequently used diagrams of the drawing (other diagrams are considered in the work of Marchuk and Yermakov) (see collector/collection [67]).

Let density of distribution of certain random variable x be given by non-negative function $p(x)$, and density of distribution of another random variable y will be $g(y)$. Further, let $y=f(x)$, $f(x)$ being the monotonic function, which for the certainty we will consider increasing. This means that for the arbitrarily taken random number ξ

for all $x < \xi$ inequality $y < f(\xi)$ is fulfilled. The probability of the fact that $x < \xi$ is accepted to call the distribution function and to express with the help of the integral

$$F(x < \xi) = \int_{-\infty}^{\xi} p(x) dx. \quad (3.3)$$

It is obvious that

$$F[y < f(\xi)] = \int_{-\infty}^{f(\xi)} g(y) dy \quad (3.4)$$

and consequently,

$$\int_{-\infty}^{\xi} p(x) dx = \int_{-\infty}^{f(\xi)} g(y) dy. \quad (3.5)$$

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If $f(x)$ monotonically decreasing function, then (3.5) will take the form

$$\int_{-\infty}^{\xi} p(x) dx = \int_{f(\xi)}^{\infty} g(y) dy. \quad (3.6)$$

Let the random variable x be evenly distributed in the interval $[0, 1]$, then $p(x)=1$ and (3.5) it is possible to rewrite in the form

$$\xi = \int_{-\infty}^{f(\xi)} g(y) dy = F(y). \quad (3.7)$$

In formula (3.7) it is possible to express y through ξ , after fulfilling inverse transformation

$$y = F^{-1}(\xi). \quad (3.8)$$

Thus, if it is possible to fulfill conversion (3.8), i.e., if it is possible to register analytical expression for $F^{-1}(\xi)$, we obtain convenient method of expression of random variable from arbitrary distribution law through another random variable from uniform distribution $[0, 1]$ ¹.

FOOTNOTE ¹. The numbers, comprised of numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 so, that in their location there is no law whatever, are called random. In order to obtain random number, for example 4-digit in interval $[0, 1]$, is necessary the number, comprised of four numerals, to multiply by 10^{-4} . ENDFOOTNOTE.

If for the solution of problem not many random numbers are required, then it is possible to take them from appropriate tables [70]. In practice they use the so-called pseudo-random numbers. These numbers in a quantity from several hundred thousand to several million (depending on program) are generated by computer itself in the process of calculation according to specific recursion relations [71]. Very fact of obtaining the sequence of such numbers according to a defined rule as if contradicts name itself "random numbers". However, such sequences are used only then, when they satisfy to all statistical to

criteria on checking of "chance" and uniformity, and therefore virtually they do not introduce systematic errors into the calculation. We used the program of the generation of pseudo-random numbers for computers BESM, given in monograph [72].

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As an example of the use of a method of inverse function let us consider the diagram of the generation of a random value y from the density of the distribution

$$g(y) = Ay^{-n}, \quad (3.9)$$

where the variable y is changed in interval of $y_0 \leq y \leq y_1$, and constant A is determined from the condition for the standardization

$$\int_{y_0}^{y_1} g(y) dy = 1.$$

In the case

$$\xi = A \int_{y_0}^y y^{-n} dy = \frac{A}{n-1} (y_0^{-n+1} - y^{-n+1}),$$

in question whence

$$y = \frac{1}{\sqrt[n-1]{y_0^{-n+1} - \xi \frac{n-1}{A}}}, \quad (3.10)$$

thus, the drawing of random variable from distribution (3.9) is reduced to the generation of random number ξ in interval $[0, 1]$ and to the calculation of fraction (3.10).

Unfortunately, by no means in all cases is it possible to fulfill inverse transformation (3.8). Thus, it is impossible to register in the analytical form inverse function for the function of the distribution of Moliere, Goudsmit-Saunderson, Landau - fundamental distributions, utilized in the problems in the passage of electrons. For these functions it is possible to find only numerical values $F(y) = \int_a^y g(t) dt$. Since the method of inverse function is comparatively simple, then it is possible to use, also, in such a case, when function $F(y)$ is assigned numerically.

Let function $F(y)$ be assigned for sufficiently large number of equidistant values y_i , which contain entire range of change y (values of functions of Goudsmit-Saunderson we they computed for 80 points in interval $0-2\pi$). Then by interpolation it is easy to find such set y_m^* , which corresponds to the sequence of the evenly distributed values of function $F(y)$, determined with the help of the relationship/ratio

$$F(y_m^*) = \frac{m}{N} \cdot \frac{1}{2}, \quad (3.11)$$

where $m=1, 2, 3, \dots, N$.

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These values y_m^* will be brought in into memory of machine. The drawing of value y is reduced now to the generation of random number ξ and the determination of the number of the cell $m(m=\xi N)$, in which the is written the corresponding value of y . If $m=\xi N$ is not whole number, then value y is located by interpolation between the values, carried

into the next arranged/located cell. The behavior of both the integral function of Goudsmit-Saunderson and of the function of the distribution of angles as a result of single scattering is such, that they rapidly approach one for the angles of scattering close to 180° . In this case it would prove to be that the equiprobable values, located in two adjacent nuclei, could correspond to two essentially distinguished angles, and the accuracy, obtained by linear interpolation, would be low. Therefore the described method it would be possible to use twice: to find the values of angles for the random numbers from 0 to 0.99 and additionally 100 values of angles for the random numbers from 0.9900 to 1.0000. If the memory of machine does not make it possible to retain this further information, it is possible to use this procedure. In the memory of machine it is brought in the sequence of unevenly distributed random numbers $F(u_i)$, corresponding N to equidistant values u_i (for example angles), and we carry out between them linear interpolation, when $F(y_i) \leq F(y_{i+1})$. In this case the accuracy in the evaluation/estimate of random value y_i will be worse $\frac{\Delta u}{2}$.

§ 3. Networks of Monte Carlo.

If we knew precise laws, which characterize reports/events of interaction of electrons with substance, then result of solution of problems by method by Monte Carlo method would depend virtually only on statistical accuracy of calculation. However, the interaction cross sections of electron with the atoms are known approximately

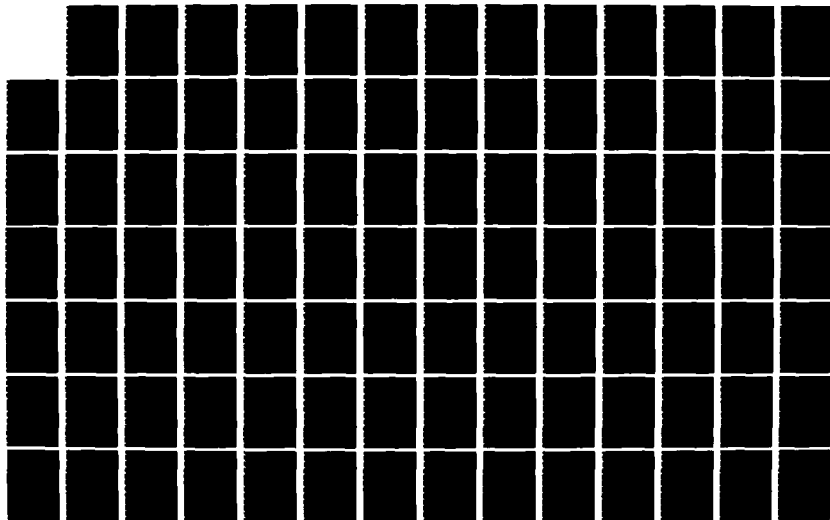
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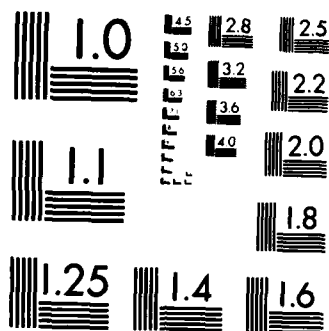
USING THE MONTE CARLO METHOD TO SOLVE PROBLEMS OF THE
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the theoretical approaches they are used its approximations/approaches, in the experimental data spread it is determined by measuring errors). Furthermore, usually is introduced an also whole series of simplifications. In particular, it is assumed that the electron simultaneously interacts only with one atom. Here the effects connected with the interaction of the electron with several atoms are not considered. This simplification makes it necessary to be bounded to the examination only of isotropic materials.

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At the same time during the research of the transfer of narrow electron beams in the thin single crystals the simplification indicated cannot be accepted, since the phenomenon of channelization discovered recently indicates the significant role of the anisotropic properties of medium. With low energy of the electrons (≤ 1 keV) it is necessary to also consider interaction with the electronic plasma, which leads to the appearance of discrete/digital energy losses.

If we assume that in problems examined/considered by us all simplifications indicated above occur, then in this case attempt to realize at trajectory with method of consecutive collisions leads to another very essential difficulty - to high expenditures of machine time. Actually, due to the high value of the interaction cross section of electron concerning the atoms for following one trajectory it would be necessary to repeat the procedure of the drawings of

10^3 - 10^4 times. After taking the time, necessary for calculating set of parameters of particle (E , \vec{r} , \vec{y}) after one interaction, equal to $100 \mu s$, we will obtain that for calculating 1000 trajectories (necessary minimum for guaranteeing the statistical authenticity of result) it will be required by 10^4 - 10^5 s, and this already too long a time. Let us note that in the problems, connected with the transfer of γ -quanta, and in the majority of neutron problems, it is necessary to trace only 20-30 collisions. For a number of reasons for the electrons it is not possible to use known procedures of the acceleration of calculation.

I. Diagram of the "amalgamated" collisions ¹.

FOOTNOTE ¹. In the literature sometimes is used the term - schematic of the "condensed" trajectories. ENDFOOTNOTE.

A way out was found in application of model of "amalgamated" collisions, i.e., when drawings are produced not after each collision from distributions, which characterize elementary report/event of scattering, but after passage of certain route segment Δt from distributions, which characterize multiple scattering of particles. Consequently, instead of the "real" particle trajectory, schematically depicted in Figure 13 (fine/small zigzag), is examined hypothetical trajectory (large/coarse zigzag). It is difficult a priori to predict, how this a replacement will affect the results of calculation. For the proof of this approach it is possible to use the

following considerations. Since elastic-scattering cross section for small angles is Rutherford, the probability of divergence of such angles will be large and, consequently, angle in multiple scattering - this the result of numerous single small-angle scatterings.

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Under these conditions it is possible to isolate a certain mean direction, along which moves the electron, and thus to fix/record segment Δt . It is clear that the less Δt , the more precisely is fixed/recorded the mean direction of electron motion and the more precisely it is possible to examine its motion near the boundaries, but in this case the time, necessary for conducting the calculations, substantially grows.

Furthermore, value Δt cannot be done less than certain minimum value, below which become inapplicable theories of multiple scattering. Thus, for example, theory of Moliere is applicable, when collision frequency in segment Δt is more than 20, i.e., for the thicknesses is more than several mg/cm². On the other hand, value Δt cannot be done very large, so as to energy losses to the ionization and the bremsstrahlung would be compared with the wave energy. Let us recall that energy losses not at all are considered in the theories of multiple scattering. Thus, the selection of length Δt to a certain extent is arbitrary, and with this fact, probably, can be connected some small, but systematic divergences, which are exhibited during the comparison of calculations with the experiment.

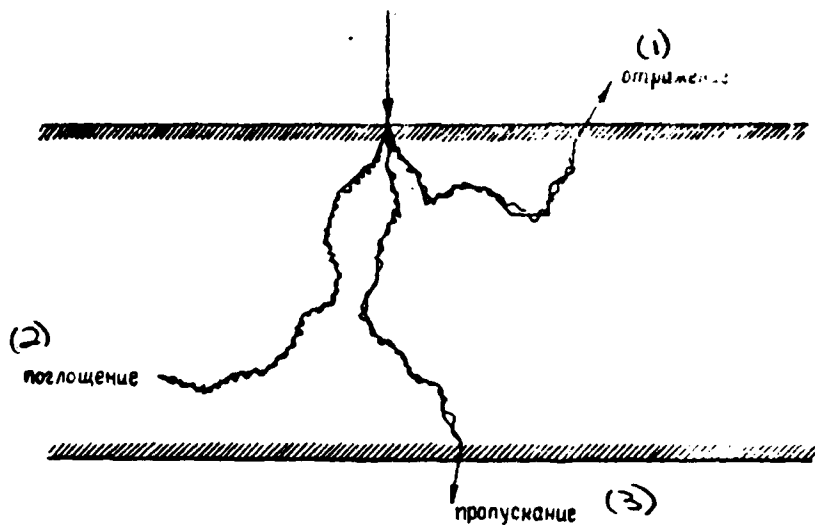


Fig. 13. Diagrammatic representation of electron paths in substance.
Key: (1). reflection. (2). absorption. (3). transmission.

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Nevertheless Δt_i is possible to select so as to reduce to a minimum these divergences.

In work of Berger [68] different methods of selection Δt_i are investigated.

1. Assume that is given energy E_i at the end of the i segment; let us require so that relative change in energy of particle in segment $\Delta t_{i+1} = t_{i+1} - t_i$ would compose always one and the same number k ($k < 1$)

$$\frac{E_i - \int_{t_i}^{t_i + \Delta t_{i+1}} \left(-\frac{dE}{dt} \right) dt}{E_i} = k, \quad (3.12)$$

where $-(dE)/(\overline{dE})^{dt}$ - average/mean energy losses, computed from formula (2.8). Assigning value of k , it is possible to compute that corresponding Δt_i .

With this method of determination Δt_i , average/mean cosine of angle of deflection proves to be in effect constant for entire sequence of segments Δt_i . Usually k choose in the form $k = 2^{-1/m}$, where m - whole number (in work [68] $m=32$). This is convenient because near the boundary of layer for a more precise evaluation/estimate of energy loss with the help of interpolation it would be possible to decrease k in n of times, i.e. to examine segments $\Delta t_i'$, determined for $k = 2^{-1/mn}$. Naturally, with this approach the consecutive account of fluctuations in energy losses is impossible and they have in mind only continuous degradation of energy by small portions $\Delta E_{i+1} = kE_i$.

2. It is possible to assume that $\Delta t_i = \text{const}$, as this is done in one of first works with use of method of Monte Carlo [74], where passage of electrons with energy in 5-55 MeV region was examined through layer of carbon. Tracking ceased, when energy of electrons reached 1 MeV.

Gradual broadening of angular distribution toward the end of landing run is deficiency in this method, which can serve as source of systematic error.

3. It is possible, finally, to assume, as we did, following work of Perkins [75] that Δt_i varies with energy in process of particle

motion

$$\Delta t_{i+1} = k E_i. \quad (3.13)$$

It is easy to see that if $-dE/dt = \text{const}$, and this is implemented virtually always, when segment Δt_i is sufficiently small so that energy losses on it would be small ($\Delta E < 0.1 E_i$), formula (3.12) is reduced to (3.13).

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Actually

$$\begin{aligned} E_i - c \Delta t_{i+1} &= k^* E_i \\ \Delta t_{i+1} &= \frac{1-k^*}{c} E_i = k E_i, \end{aligned}$$

where k^* - this coefficient k in formula (3.12), and $-dE/dt = c$. In order to use formula (3.13) for any energies of electrons, we consider coefficients of k as energy- dependent.

Table 8 gives values $k=k(E)$, which are used in our calculations. This method of selection Δt_i makes it possible to consider fluctuations in energy losses and to examine, consequently, is more fully the transfer of electrons in the substance.

According to Berger's diagram ' after determination Δt_i , goes drawing of scattering angle from distributions of Moliere or Goudsmit-Saunderson, moreover input energy for obtaining this distribution is energy E_i , which electron in the beginning of segment had.

$$E_i = (k)^i E_0, \quad i = 0, 1, 2, \dots \quad (3.14)$$

FOOTNOTE ¹. Apparently, Berger uses a diagram taking into account fluctuations, but it is nowhere published. ENDFOOTNOTE.

Transition/junction into the necessary reference system is produced with the help of the formulas of spherical trigonometry. Azimuth angle is assumed to be that distributed evenly in the interval $0-2\pi$, and its random value is realized with the help of the further drawing. Cartesian (or other) coordinates of the salient points of electron path simultaneously are calculated. The sequence of the stages of calculation according to our diagram is such: 1) for this energy of electron we choose $k(E_i)$ from Table 8; 2) with the help of coefficient of k we find $\Delta t_i = kE_i$; 3) we compute Δ_i and ΔE_i for current energy E_i according to formula (2.19).

Table 8.

E, MeV	6÷4	4÷2	2÷0,8	0,8÷0,3	0,3÷0,05
k	0,1	0,05	0,03	0,02	0,01

Key: (1). MeV.

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Methodology of drawings from integral function of distribution of Simon (Landau) for energy losses only to ionization (see Table 6 of application/appendix) is analogous this described in § 2 of the chapter. Let us note that the distribution function was approximated by curve $\frac{0,8463}{\left(\frac{\Delta E_n - \Delta E_s}{\Delta_0}\right)}$ for all $\frac{\Delta E_n - \Delta E_s}{\Delta_0} > 10$. After finding $\frac{\Delta E_n - \Delta E_s}{\Delta_0}$, we compute the loss of energy ΔE_n . For all $E_i > 0,511$ MeV through formula (2.11) we find energy losses to the bremsstrahlung in segment Δt_i . We compute the total losses of energy $\Delta E_{n1} = \Delta E_{n1} + \Delta E_{T1}$ and we find energy of electron E_{i+1} at the end of segment Δt_i .

$$E_{i+1} = E_i - \Delta E_{n1}.$$

Upon consideration of fluctuations in energy losses to bremsstrahlung drawing of total losses of energy is produced from the tabulated distributions and those input into memory of computer (2.26) for entire sequence of energies and the connected with them Δt_i . In this case, for current energy E_i is chosen the distribution, designed for the value of the energies nearest to E_i . Reaching/achievement of certain minimum value E_{min} , which is assigned by researcher and is fixed/recorded in the memory of machine, is the first criterion of the

cessation/discontinuation of the tracking of this trajectory. In our calculations $E_{min} = 0.02 \div 0.05$ MeV. Electron energy $E > E_{min}$ is considered to be absorbed directly in the vicinity of the point, where this condition is satisfied, and the tracking of new trajectory begins, if $j < N$, where N - number of planned for the tracking trajectories. If $E < E_{min}$, we pass then to the drawing of scattering angle. The logic circuit of the calculations of energy of electrons is shown in Figure 14.

4. Schematic of drawing of scattering angle θ_{i-1} , calculated off direction electron motion in segment Δt_i , it is shown in Figure 15. In this case on input energy E_i is found out the nearest energy, for which the integral distribution of Moliere or Goudsmit-Saunderson is known, and very methodology of drawing is analogous described above for the distributions, assigned numerically. For the transition/junction into the reference system, connected with some specific direction, for example, by the positive axis OZ, are used the formulas of spherical trigonometry

$$\cos \psi_i = \cos \psi_{i-1} \cos \theta_i + \sin \psi_{i-1} \sin \theta_i \cos \chi_i. \quad (3.15)$$

Angles $\psi_i, \psi_{i-1}, \theta_i, i=1, 2, \dots, n$ are shown in Figure 16. Azimuth angle χ_i is distributed equiprobably in the interval $0-2\pi$, if we disregard/neglect polarizational effects. The schematic of the drawing of this angle is trivial: random number ξ is chosen and it is computed $\chi = 2\pi\xi$.

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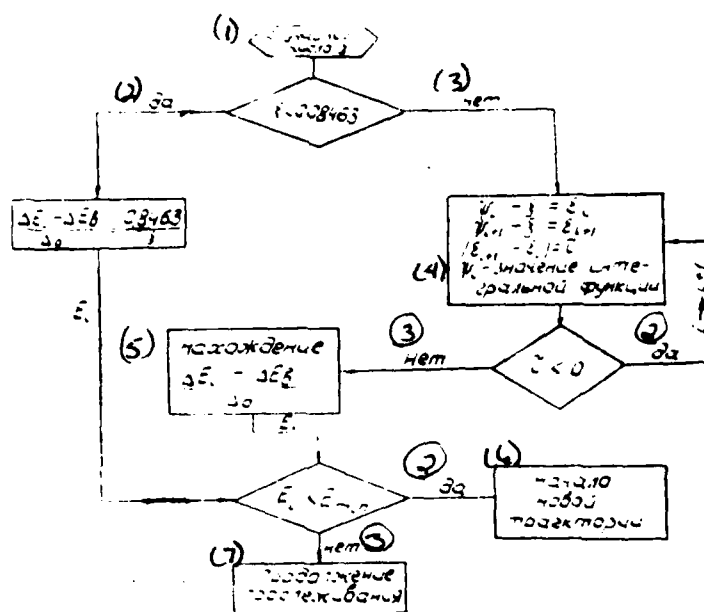


Fig. 14. Logic circuit of calculations of energy of electron at the end of segment.

Key: (1). random number ξ . (2). yes. (3). no. (4). value of integral function. (5). Determination. (6). Beginning of new trajectory. (7). Continuation of tracking.

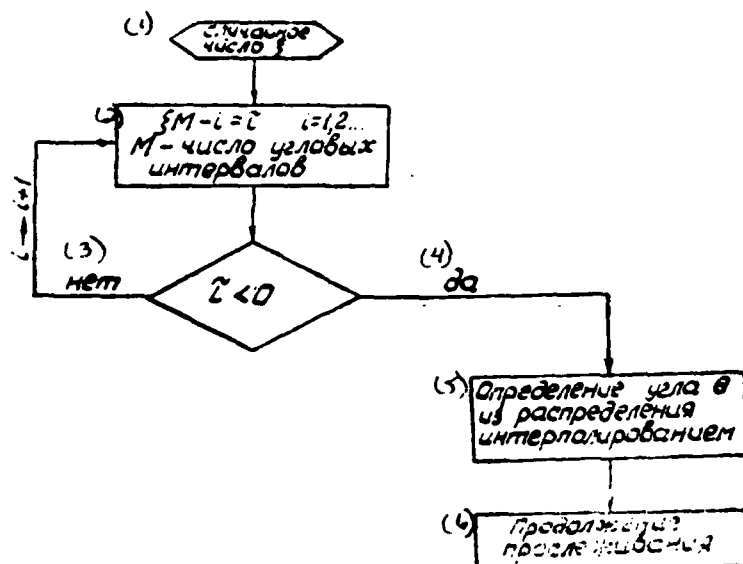


Fig. 15. Logic circuit of calculated angle in multiple scattering.

Key: (1). Random number ξ . (2). number of angular intervals. (3). no. (4). yes. (5). determination of angle θ from distribution by interpolation. (6). Continuation of tracking.

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Calculation $\cos \chi$ we produced not on standard program for this computer ¹, but we used the table of $\sin y$ ($y=0^\circ \div 90^\circ$), the registered into the memory machine with a space of 1° .

FOOTNOTE ¹. There is an efficient diagram of Monte Carlo [76], which uses the methodology of exception/elimination, which makes it possible to obtain immediately $\cos \chi$ and $\sin \chi$, using on the average 2, 6 random numbers. ENDFOOTNOTE.

Turning to this table occurs in the dependence on ξ through the functions

$$\begin{aligned} & \sin\left(\frac{\pi}{2} - 2\pi\xi\right), \xi \leq 0,25, \\ & -\sin\left(2\pi\xi - \frac{\pi}{2}\right), 0,25 < \xi \leq 0,5, \\ & \sin\left(\frac{3\pi}{2} - 2\pi\xi\right), 0,5 < \xi \leq 0,75, \\ & \sin\left(2\pi\xi - \frac{3\pi}{2}\right), 0,75 < \xi \leq 1. \end{aligned} \quad (3.16)$$

5. Cartesian coordinates of consecutive salient points of trajectory are computed from relationships/ratios

$$z_i = \sum_1^i \Delta t_i \cos \psi_{i-1}, \quad (3.17)$$

$$\begin{aligned} x_i &= \sum_1^i \Delta t_i \sin \psi_{i-1} \cos \varphi_i, \\ y_i &= \sum_1^i \Delta t_i \sin \psi_{i-1} \sin \varphi_i \end{aligned} \quad (3.18)$$

($x_0=y_0=z_0=0$ - coordinate of impact point in electron). Angles included in (3.18), are located with the help of the formulas

$$\begin{aligned} \sin(\varphi_{i+1} - \varphi_i) &= \frac{\sin \theta_i \sin \chi_i}{\sin \psi_{i+1}}, \\ \cos(\varphi_{i+1} - \varphi_i) &= \frac{\cos \theta_i - \cos \psi_i \cos \psi_{i+1}}{\sin \psi_i \sin \psi_{i+1}}. \end{aligned} \quad (3.19)$$

It is assumed that $\varphi_0=0$. Since the majority of problems is solved in the one-dimensional case, only coordinate z , computed from (3.17) interests us.

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For spherical geometry (Fig. 17) tracking electron occurs not on Cartesian coordinates of end/lead of segments Δl_i , but on radius-vector R_i , carried out into end/lead of this segment. In this case the reference system of angles is conveniently connected with radius-vector R_i . It is not difficult to establish/install simple correlations for determination R_i and ψ_i

$$R_{i+1}^2 = R_i^2 + \Delta l_{i+1}^2 - 2\Delta l_{i+1} R_i \cos \psi_i, \quad (3.20)$$

$$\cos \psi_{i+1} = \cos \theta_{i+1} \cos \alpha_{i+1} + \sin \theta_{i+1} \sin \alpha_{i+1} \cos \chi_{i+1}. \quad (3.21)$$

Values $\cos \alpha_{i+1}$ and $\sin \alpha_{i+1}$ in (3.21) can be found from the expressions:

$$\frac{R_{i+1}}{\sin \psi_i} = \frac{R_i}{\sin \alpha_{i+1}} \quad (3.22)$$

and

$$\cos \alpha_{i+1} = \frac{R_i \cos \psi_i}{R_{i+1}} - \frac{\Delta l_{i+1}}{R_{i+1}}. \quad (3.23)$$

Very procedure of drawings of energy and angle, and also other operations, implemented in process to tracking under conditions of spherical geometry, remain the same as in the case of plane geometry.

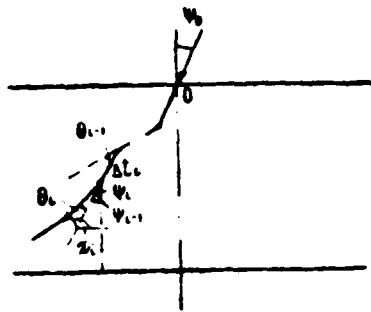


Fig. 16.

Fig. 16. Connection/communication between scattering angles in consecutive stages of calculation for plane geometry.

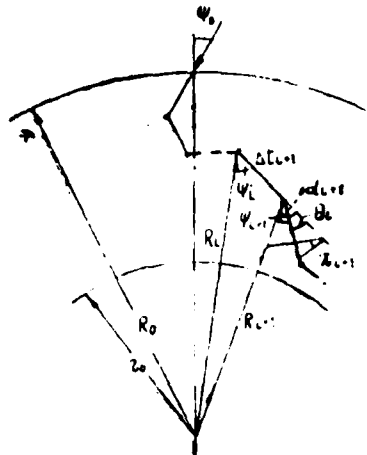


Fig. 17.

Fig. 17. Connection/communication between scattering angles in consecutive stages of calculation for spherical geometry.

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6. After finding coordinates of end/lead of segment, we clarify, did not occur intersection with one of boundaries interesting us.

For simplicity let us consider one-dimensional problem. We check $z = z_0^{FP}$ ($z_0^{FP} = z_0 = 0$), did not occur the intersection of boundary in the direction of reverse/inverse hemisphere. If $z \leq z_0^{FP}$, electron is considered back-scattered, and its energy and angle after the appropriate refinements are fixed/recorded in the specific energy and angular ranges; tracking ceases, and new trajectory begins. If $z > z_n^{FP}$ (z_n^{FP} - boundary within the sample), then the intersection of this

boundary occurred. Characteristics (energy, angle), with which the electron intersects z_n^{rp} , will be input into the storage cells of computer corresponding to this layer, tracking is continued, until electron is absorbed in substance ($E = E_{min}$) if it will not cross boundary z_N^{rp} of sample, which separates/liberates it from the vacuum. It is necessary to now consider the method of calculating the characteristics of electron outgoing from the boundary. Distance of boundary Δt_{i+1} we find from the formula

$$\Delta t'_{i+1} = \left| \frac{z_n^{rp} - z_i}{z_{i+1} - z_i} \right| \Delta t_{i+1}, \quad (3.24)$$

where z_{i+1} and z_i - coordinate of the ends/leads of segment Δt_{i+1} . Strictly speaking, we must play values E'_{i+1} , θ'_{i+1} at the end of the segment from the appropriate distributions as this done above. However, if E'_{i+1} is easy to compute on the basis of the universal distribution of Simon, then for drawing θ'_{i+1} it was necessary to have a distribution of angles in multiple scattering, designed for $\Delta \tau'_i$. This would lead to the further charging of the memory of machine. For the approximate determination of angles and energies linear interpolation was used. Since the checking to the intersection of boundaries is produced after that how is done next space Δt_{i+1} , we know set of parameters E_i , ψ_i , z_i in the beginning of segment and E_{i+1} , ψ_{i+1} , z_{i+1} at the end of it, then

$$E'_{i+1} = E_i + \frac{\Delta t'_{i+1}}{\Delta t_{i+1}} (E_{i+1} - E_i) \quad (3.25)$$

and

$$\psi'_{i+1} = \psi_i + \frac{\Delta t'_{i+1}}{\Delta t_{i+1}} (\psi_{i+1} - \psi_i)$$

In contrast to this methodology Berger [68] calculates energy E_{i+1} from average/mean losses in segment Δt_i . Undoubtedly, both approaches lead to the errors in the angular and energy distributions of the outgoing electrons, but they, apparently, they are small, since the results of calculations will agree well with the experiment. One should note that tracking after the intersection of boundaries

$z_n^{rp} (z_n^{rp}, z_N^{lp})$ is continued with sets $E_{i+1}, \psi_{i+1}, z_{i+1}$, recorded in the computer memory, which are obtained correctly.

II. Diagram of of "Successive" Collisions.

Fundamental results of applying Monte Carlo method to transfer of electrons are obtained according to diagram presented above of "amalgamated" collisions. Nevertheless several works were carried out according to the diagram of "consecutive" collisions. The target of such calculations, which are implemented only for the very thin samples, to investigate the possible systematic errors, introduced by the diagram of the "amalgamated" collisions, and to explain the effect of interaction cross section on the characteristics, which especially sensitive to its form, for example, scattering to the large angles or to the energy losses in the layer of substance. This program is formulated in the works of MacCallum [77] and Paul and Tatzber [78]. Somewhat later, Hara [79] fulfilled the analogous calculations even for a specific problem - the evaluation/estimate of heat release in the window, through which the electron beam is withdrawn from the accelerator. The logic circuit of the tracking of trajectory,

borrowed from work [79], it is shown in Figure 18. The drawing of the landing run between collisions is produced from the distribution

$$P(x, E)dx = \mu_n \exp(-\mu_n x) dx, \quad (3.26)$$

where coefficient μ_n is connected with total cross section $\Sigma_n(E)$ with the relationship/ratio

$$\mu_n = \sum_{i=1}^N \Sigma_i(E) \text{ (cal/cm}^2\text{)}. \quad (3.27)$$

In [80] the effective procedure of drawing from this distribution is described.

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Depending on energy range in question total cross section is composed of sections of processes of 1) elastic scattering; 2) inelastic scattering; 3) bremsstrahlung, etc. In the following drawing the form of process from the distribution

$$p_1 = \frac{\mu_1}{\mu_n}, p_2 = \frac{\mu_2}{\mu_n}, \dots, p_n = 1 - p_1 - \dots - p_{n-1}. \quad (3.28)$$

is clarified.

For this random number ξ is equal consecutively/serially with relations (3.28), and if, for example, $p_1 < \xi \leq p_2$, then we have process 2 and, etc. Expressions for $\mu_1, \mu_2, \dots, \mu_n$ are obtained by the integration of the corresponding expressions for the differential cross sections. During the calculation of μ_1 (elastic collisions) in work [79] Rutherford section is undertaken.

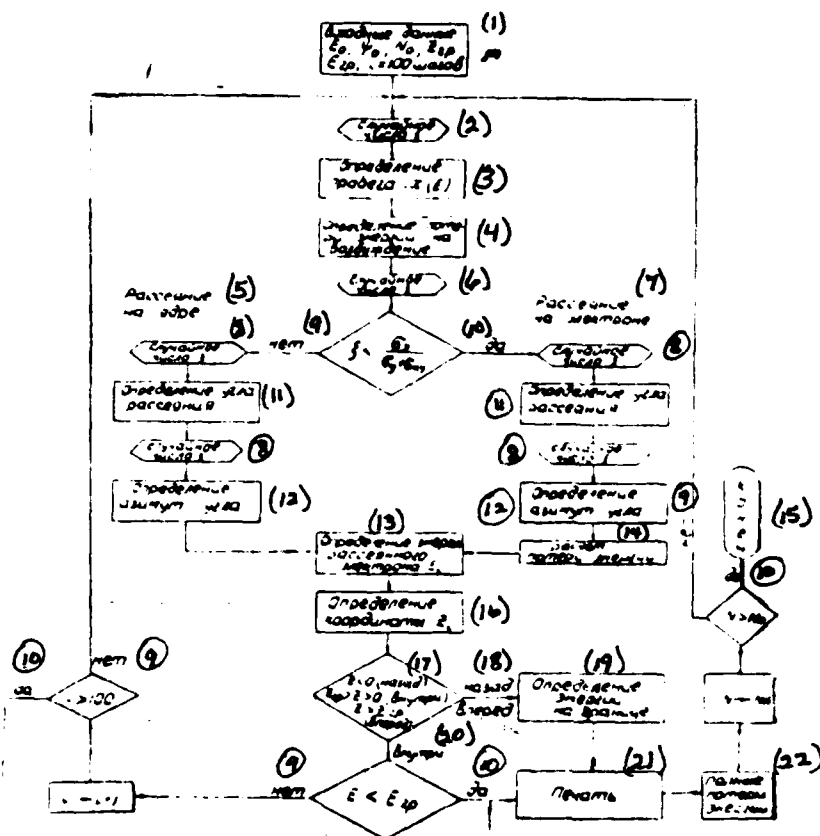


Fig. 18. Logic circuit of tracking of electron by "consecutive" collisions.

Key: (1). input data ^{steps} (2). random number. (3). determination of path (4). determination of energy loss to excitation. (5). Scattering on kernel. (6). random number. (7). Scattering on electron. (8). random number. (9). no. (10). yes. (11). determination of scattering angle. (12). determination of azimuth angle. (13). determination of energy of stray electron:. (14). calculation of energy loss. (15). end/lead. (16). determination of coordinate (17). ... (backward) ... (inside):. (forward). (18). backward. (18a). forward. (19). determination of energy on

boundary. (20). Inside. (21). Printing. (22). total losses of energy.

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Obviously, there are no fundamental difficulties whatever for the calculation μ_1 , if we use the tabulated values of the section of Mott. Inelastic electron-electron scattering was described by Moller's formula (1.20). For this process μ_2 it is computed from formula (1.25).

After explaining form of process, we find energy or angle of stray electron from integral distributions

$$P(\theta) = \frac{\int_{\theta_{\min}}^{\theta} \frac{d\sigma}{d\Omega} d\Omega}{\int_{\theta_{\min}}^{\theta_{\max}} \frac{d\sigma}{d\Omega} d\Omega}, \quad (3.29)$$

$$P(E) = \frac{\int_{E_{\min}}^E \frac{d\sigma}{dE} dE}{\int_{E_{\min}}^{E_{\max}} \frac{d\sigma}{dE} dE}, \quad (3.30)$$

where θ_{\min} , E_{\min} , θ_{\max} , E_{\max} - minimum and maximum angles and energy of electrons after scattering.

The manifestation of the minimum angles is caused by divergence of section at $\theta=0^\circ$ (for example, Rutherford without shadowing), and minimum energies - by process of exciting electron shell. The account of energy loss to the excitation is produced on the average/mean losses, using formula (2.3). For $\frac{d\sigma}{d\Omega}$, $\frac{d\sigma}{dE}$ it is possible to use the expressions for the differential cross sections, given in chapter I.

Since energy and angle are connected, then, after playing of distributions (3.29) or (3.30) one of these parameters, it is easy to find another. In the case of elastic scattering usually gaming process is conducted from (3.29), and they disregard energy losses (scattering on the heavy nucleus).

After computing scattering angle, we give it into necessary reference system according to formula (3.15) and determine coordinates of particle. As can be seen from diagram in figure 18, in it are used the same criteria of the break of trajectory as in the diagram of the "amalgamated" collisions. The results of the calculations, carried out by the authors of the cited works, are sufficiently plain. In work [78] with accuracy of $\pm 20\%$ the agreement with the data of Berger [68] for the back-scatter factors is obtained. A good agreement of these calculations with the experiment according to the coefficient of transmission T_N is noted also. More interesting information is obtained in work [79]. The calculated values of absorbed energy are in a good agreement with experiment [81]. In Table 9 we compare these data with the data of our calculations for Al.

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As can be seen from Table 9, there is good agreement between calculations. This can be considered as confirmation of the applicability of the model of the amalgamated collisions generally and even in the region of such thicknesses 1 , where the criteria of the applicability of the theory of multiple scattering strictly are not

implemented (see latter/last column Table 9).

FOOTNOTE 1. However, in this region the calculation procedure ⁴Jara [79] should be considered as the version of the diagram "catastrophic" collisions. ENDFOOTNOTE.

Belyayev and Krupman [82] carried out calculation of angular distribution of electrons, scattered in very thin layer of substance. The drawing of landing runs in this case was produced from the Poisson distribution. A good agreement with the experiment is obtained.

In considered diagram, the use of different methods of accelerating calculation would raise speed and efficiency of calculation. For example, the correlated tests (see below) would make it possible to consider the validity of one or the other methods of the account of the shadowing and other theoretical prerequisites/premises. Conducting such calculations is of undoubted interest.

III. Diagram "catastrophic" collisions.

In this diagram, proposed by Schneider and Cormack [83], such rare events, as scattering to large angle $\theta > \theta_m$ or large relative loss of energy $w > w_m$, are called "catastrophic" and are examined separately. Particle trajectory consists of the segments, within which there are no catastrophic collisions, and the tracking of particle is produced

according to the usual diagrams: by Berger, ours, etc. The most "catastrophic" collisions occur only at the ends/leads of the segments.

Table 9.

E, Мэв (1)	(1) ΔE , Мэв						(3) Число со- ударений в 100 мкм
	100 мкм (2)		200 мкм (2)		300 мкм (2)		
	[79]	наши расчеты (4)	[79]	наши расчеты (4)	[79]	наши расчеты (4)	
0,4	0,063	0,072	0,165	0,185	0,299	0,288	8,07
0,7	0,062	0,048	0,112	0,122	0,192	0,224	2,77
1,0	0,062	0,049	0,102	0,100	0,168	0,170	1,66

Key: (1). MeV. (2). μm . (3). Collision frequency in 100 μm .
(4). our calculations.

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The sequence of the states of particle is written/recorded with the help of the matrix/die

$$E_0, E_1, E_1^*, E_2, E_2^*, \dots, E_i, E_i^*, \dots$$

$$r_0, r_1, r_1^*, r_2, r_2^*, \dots, r_i, r_i^*, \dots$$

$$u_0, u_1, u_1^*, u_2, u_2^*, \dots, u_i, u_i^*, \dots$$

The set of parameters E_i, r_i, u_i and E_i^*, r_i^*, u_i^* characterizes the state of particle respectively to the i-th and immediately after the i-th "catastrophic" collision.

Diagram of drawings is such:

1. Value of Q is determined from the distribution

$$P(Q) dQ = e^{-Q} dQ. \quad (3.31)$$

2. Knowing E_i^* , we find E_{i+1} (energy to i+1 catastrophic collision) from formula

$$Q = \int_{E_{i-1}}^{E_i^*} \frac{dE}{\Lambda_m} \left(- \frac{dE}{dx} \right)^{-1}, \quad (3.32)$$

where Λ_m - landing run with respect to loss of energy $w = w_m$, and $\left(- \frac{dE}{dx} \right)$ - average/mean energy losses, computed from formula (2.7). In the case of electron-electron scattering $\Lambda_m = \Lambda_m(u_m)$, where $\Lambda(u_m)$ is obtained from (1.25) by substitution w_m instead of w .

3. Knowledge E_{i-1} and E_i^* makes it possible to find value of segment Δt_i between i and $i+1$ "catastrophic" collisions.

4. Rate of energy loss in "catastrophic" collision is found with the help of drawing from distribution

$$P(w = w_m) = \int_{w_m}^{\omega} \frac{dz}{dw} dw \Big/ \int_{w_m}^{\omega} \frac{dz}{dw} dw, \quad (3.33)$$

where $\frac{dz}{dw}$ is determined by formula (1.20).

Using played value of w , we find energy of stray electron E_i^* and energy $E_i = \delta \cdot \text{elektrona } E_i^* = (1 - w) E_i^*$

$$E_i = w E_i^*$$

With the help of (1.21) and (1.22) it is easy to find new directions of motion of both electrons. Transition/junction into the necessary reference system is completed employing usual procedure.

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In the extreme case, when $w_m = \frac{1}{2}$, diagram "catastrophic" collisions passes into diagram, where are considered only continuous

losses of energy ($\nu_m = 0$). In other limiting case, when $w_m \rightarrow 0$ (mean free path between the collisions it becomes equal to 0), all collisions are "catastrophic", i.e., this diagram passes into the diagram of consecutive collisions.

Thus, diagram "catastrophic" collisions occupies intermediate position between diagrams examined above. This makes it possible to hope that during the proper development it will be possible to optimally combine advantages of both maximum diagrams in it. Even now are clearly visible certain the valuable sides of the diagram: the more correct account of fluctuations in the losses of energy (drawing it is conducted only for the large losses) and obtaining the detailed information about the energy and angular distribution of δ -electrons. In the future its application for the study of backscattering is possible.

Recently diagram "catastrophic" collisions was used by Kolchuzhkin and Shevtsova [84]. They showed that the results of calculations according to this diagram will agree well with the experiment, and they investigated the methods of evaluation/estimate ν_m and E_m . To the diagram "catastrophic" collisions examined closely borders on Meissner's diagram [85], in which is done the first attempt to combine in one calculation the analytical methods, based on the solution of the equation of transfer, and the Monte Carlo method. This combined methodology has already been used in transport problem of γ -radiation [86].

In calculation of Meissner small-angle scattering and small energy transfers are examined with the help of equation of transfer, and large energy losses with the help of Monte Carlo method. Probably, this methodology will make it possible to solve many difficult questions of theory of transfer, in particular the account of fluctuations in energy losses. Meissner calculated in the barrier geometry of the characteristics, which are obtained in the routine calculation by Monte Carlo: the coefficients of the transmission and the reflections, energy spectra and distribution of absorbed energy. Our calculations satisfactorily will agree with Meissner's results, with exception of the distribution of the absorbed energy. Meissner's methodology is attractive, since in it it is possible to use advantages of analytical approach. However, in it there are still many vaguenesses, including how to make more precise diagram for the case of the heavy substances, when radiation losses become essential.

In conclusion we give Table 10, in which in chronological of order are equal versions of basic concepts of "amalgamated" collisions, used by different authors.

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Table 10. Versions of the diagrams of the "amalgamated" collisions.

(1) Автор	(2) Потери энергии на ионизацию	(3) Потери энергии на тормозное излучение	(4) Угловое распределение	(5) учет вторичного излучения	(6) Геометрия	(7) Использование методов ускоре- ния счета
Sidei et al. [87]	(8) Средние	—	(9) Мольера	—	(10) Одно измерение	—
Leiss et al. [74]	(8) Средние	(11) С учетом флук- туаций по Гейт- леру	(12) Гауссовское	—	(15) Одно измерение	—
Berger [68]	(13) Непрерывные	—	(14) Мольера. Гаудсмита- Саундерсона	—	(15) Три измерения	(16) Коррелированные испытания
Perkins [75]	(17) С учетом флукту- аций по Ландау	—	(18) Мольера	—	(15) Одно измерение	—
Mar [83]	(19) Непрерывные	(11) С учетом флук- туаций по Гейт- леру	(18) Мольера	—	(15) Одно измерение	—
(20). Наша схема [89]	(21) С учетом флук- туаций по Симо- ну	(22a) Средние потери (22b) Комбинирован- ное распределе- ние	(23) Гаудсмита- Саундерсона	(24) Аналитиче- ски	(15) Одно измерение. сфериче- ский слой	(25) Расщепление - «ру- летка» коррелиро- ванные испытания
(26) Баранов и другие [90]	(19) Непрерывные	—	(18) Мольера	—	(15) Одно измерение	—

Key: (1). Author. (2). Energy losses to ionization. (3). Energy losses to bremsstrahlung. (4). Angular distribution. (5). Account of secondary radiation. (6). Geometry. (7). Use of methods of accelerating calculation. (8). Average/mean. (9). Moliere. (10). One measurement. (11). Taking into account fluctuations according to Heitler. (12). Gaussian. (13). Continuous. (14). Moliere, Goudsmit-Saunderson. (15). Three measurements. (16). Correlated tests. (17). Taking into account fluctuations according to Landau. (18). Moliere. (19). Continuous. (20). Our diagram [89]. (21). Taking into account fluctuations according to Simon. (22a). Average/mean losses. (22b). Combined distribution. (23). Gaudsmit-Saunderson. (24). Analytically. (25). Splitting/fission + "tape

measure" correlated tests. (26). *Baranov* Monitor lizards and others [90].

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§ 4. Methods of accelerating the calculation.

It is known that success of application of Monte Carlo method to problems of transfer of γ -quanta and neutrons to a great extent is connected with use of methods of accelerating calculation. We have in mind both the optimum organization of the program of calculations and the special procedures which ensure the efficiency of drawings, in such a case, when in the direct tests only the insignificant part of the histories carries the information interesting us.

Analysis of these procedures was given in surveys/coverage of Kahn [76], Kalos and Goertzel [91], in monographs [60, 64, 65], in collectors/collections [67, 68] and in numerous articles of colleagues of Novosibirsk data processing center. Let us note that many procedures developed for γ -quanta and neutrons, can be transferred easily to the electronic trajectories in the diagram of consecutive collisions. Since this diagram thus far still rarely is used, we will consider the possible methods of the acceleration of calculation in application to the diagram of the "amalgamated" collisions. First of all let us formulate several general/common recommendations regarding the optimum organization of program for computers.

It is clear that

1. \wedge that the procedure of drawings, repeated, must be simple.

This it is possible to achieve by the preparation for initial

distributions in the form of numerical matrices/dies by the method, described in §2 of this chapter. It is generally necessary to attempt to bring in into the memory of machine all distributions and other dependences, to which it is necessary to be converted during gaming procedure.

2. Algorithm and program must be organized so as to ensure simultaneous calculation of all points of unknown dependence. Thus, in the problem, in which is investigated the dependence of the coefficient of transmission on the thickness of the sample, it is necessary to consider simultaneously the passage of the electrons through several intermediate boundaries. In this case besides noticeable gain in the time it is possible to decrease the statistical fluctuations due to the correlation of the events, which occur at adjacent points.

3. Information obtained as a result of calculation, must require minimum further processing. If this information will be used in further calculations, it is convenient to put out it immediately on the punch cards.

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Large time gain can be obtained also by application of special criteria, which make it possible to break "uninteresting" trajectories (in sense of interesting us information). For example, in the problem of backscattering it is clear that only the electron, which penetrated

at a depth less than its residual range $R_0(E)$ (E -energy of electron at this depth), has the capability to leave the sample into the rear hemisphere. Consequently, after introducing into the memory of machine the table of values of the path-energy, we can cease the tracking of electron in the case, "unfavorable" for the backscattering. The sequence of applying this criterion is shown schematically in Figure 19. Checking is produced immediately after the drawing of the new value of energy of electron and checking $E \geq E_{min}$. This criterion must be used in the diagram of the output of electron into the forward half sphere. In the problems, in which us interests the absorbed energy in the layer, electronic trajectory must be traced in practice to the end/lead, and the criterion of cessation/discontinuation is applicable only to layer $d(d \leq d_{max})$. In this case into the memory are introduced several values E_d (according to a number of layers, if they different thickness) or one value E_d - the energy, necessary for electron for the passage of layer d .

In Figure 20 it is shown how to use this criterion. Efficient proves to be the statistical method of the break of trajectory, known by the name "roulette" [66]. Its essence consists in the fact that with probability p the electron is tracked, and with probability $1 - p$ tracking terminates. For obtaining the unbiased estimate the weight of the particle, "which survived" after the application of a "tape measure", increases from 1 to $1/p$. "tape measure" has sense to use in such a case, when analytical criterion is not yet applicable, but

trajectory does not contain necessary information, for example electron moves in parallel to boundaries. As far as the selection of probability p is concerned, its evaluation/estimate usually is produced from the condition of obtaining the minimum of expression $D[\xi] \cdot t$, where $D[\xi]$ - dispersion, t - time of calculation.

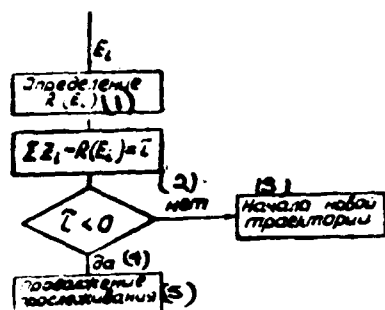


Fig. 19. Logic circuit of use of the criteria of cessation/discontinuation of tracking of trajectory on leaving of electron from sample.

Key: (1). determination. (2). no. (3). It began beam of trajectory. (4). yes. (5). continuation of tracking.

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Frequently "tape measure" uses together with methodology of splitting/fission (branching [66]. This methodology, used by us for the first time for the electrons in the problem of backward scattering [92], consists of the following: electron, for the first time turned back/ago (Fig. 21), is split into n of the trajectories, each of which has a weight $1/n$ and subsequently is outlined independently. The intersection of the assigned boundaries of layers i during the motion to the surface of barrier leads to splitting/fission n_i , moreover the parameters of particles (angle, energy, coordinate) at the points of splitting are memorized. The tracking of each of the split trajectories is conducted until the particle exits from the barrier or before the absorption in it. The diagram, used us for the

memorization of the parameters of particles, and also the procedure of consecutive return to the points of splitting/fission up to first point of rotation, is generalization of the lexicographical process, developed in the work of Golenko [71]. Since we are only interested in the back-scattered particles, tracking of the particles, which turned into the depth of barrier after splitting/fission, is limited with the help of the "tape measure", moreover for p_i was selected value $p_i = \frac{1}{n_i}$. The use of this methodology makes it possible seemingly to draw out to the surface the electrons, scattered in the depth of barrier and which have small probability of exiting leave into the rear hemisphere. Table 11 gives the results of the calculations of back-scatter factor for different orders of splitting.

Order of splitting/fission -sequence of values n_i ($i=1, 2, 3...6$ in our case).

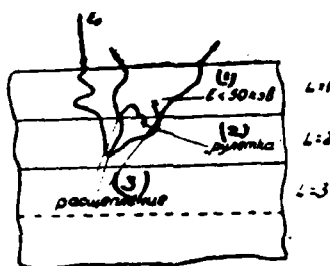
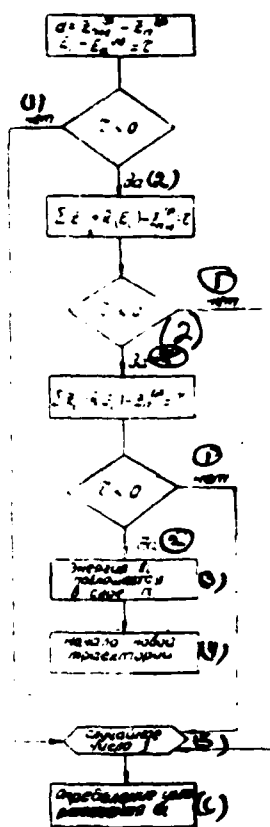


Fig. 20.

Fig. 21.

Fig. 20

Logic circuit of the use of a criterion of the
cessation/discontinuation of the tracking of trajectory in this layer.

Key: (1). no. (2). yes. (3). energy ... is absorbed in layer.
(4). in roan new trajectory. (5). random number. (6).
determination of scattering angle.

Fig. 21. Schematic diagram of application of methods of splitting/fission and "tape measure".

Key: (1). keV. (2). tape measure. (3). splitting/fission.

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If the first rotation occurred, for example, in the second layer, a number of splittings/fissions is taken by the equal to 2 and, etc. The efficiency of this calculation procedure is evaluated according to minimum $D[\xi] \cdot t$. It is reached at the order of splitting/fission by 3:2:1:1:1:1. Gain in comparison with the calculation without the splittings/fissions composes ~35%. More careful research of methodology carried out in the work Ogibin (see collection [67]).

Methodology of correlated tests, which consists in use of one and the same set of histories for research of two close processes, for example transfer of electrons and positrons, proves to be simple and efficient. Since scattering cross sections for both particles barely differ from each other, the with the limited number of independently played trajectories difference between the coefficients of transmission can and not be showed due to the large dispersion of calculation. Let we seek a difference in the coefficients of the transmission δ of positrons T_N^+ and electrons T_N^- , $\delta = T_N^+ - T_N^-$. Then the dispersion of difference can be evaluated with the help of the formula $D[\delta] = D[\xi] - 2\rho \sqrt{D[\xi] D[\eta]} + D[\eta]$, (3.34) where $D[\xi]$ and $D[\eta]$ - dispersion in evaluation/estimate T_N^+ and T_N^- , and ρ - correlation coefficient. When $\rho \approx 1$, then the dispersion of difference becomes small. In our case electronic and positron trajectories are actually strongly correlated, since entire difference between them consists in the small shift of the maximum of the angular distribution of the positrons to the side of smaller angles. The energy-loss distributions are identical. We used this methodology for the systematic study of difference in the transmission of positrons and electrons [93].

Table 11.

$$E_0 = 0,6 \text{ Мэв}, Z = 13, \varphi_0 = 0^\circ$$

(1) Порядок рас- щепления	(2) Число траекто- рий	(3) Число частиц в отражен. спектре	R_N %	R_E %	(4) $D[\frac{1}{2}]$ (относит. едини- цы)
5) 1:1:1:1:1 (без расщепле- ния)	2500	200	8,0 ± 0,6	3,8	45
2:2:2:1:1:1	500	210	9,3 ± 1,3	4,9	39
3:3:3:1:1:1	325	358	9,5 ± 1,6	5,0	56
3:2:1:1:1:1	500	286	8,8 ± 1,3	4,4	35
1:2:3:1:1:1	550	142	8,8 ± 1,2	4,5	42

Key: (1). Order of splitting/fission. (2). Number of trajectories. (3). Number of particles in it is reflected. spectrum. (4). (rel. unit). (5). ... (without splitting/fission).

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Study of effect of angle of incidence on characteristics of electrons, passing through plane layer of substance, is another important application of methodology of correlated tests. In this case it is necessary to carry out parallel calculation according to our diagram for several angles of incidence, using a set of those played one time of parameters E_0, θ_0 , i.e. to actually turn trajectory around point $x_0=0, y_0=0, z_0=0$. Similar methodology was used for neutrons in the work Dyad'kin et al. [94].

Experiment of application of Monte Carlo method showed that it is hardly expedient to create one program for solution of many problems of transfer of electrons. In our practice we use several programs, it is sufficient general/common in order to obtain the maximum volume of

information, but it is at the same time sufficient narrow, in order to for certain circle of the problems of detecting/exposing all reserves for the acceleration of calculation. The characteristics of these programs are given to [95].

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Chapter IV.

Results of calculations.

In the present chapter are presented results of calculations, carried out by Monte Carlo method, given are data for coefficients of transmission and and reflection, tables of spectra after barriers of different thickness, distribution of absorbed energy in plane layers. These data are suitable for the interpolation in the examined region of energy of electrons 0.4-6.0 MeV and angles of incidence of 0-81°. Are described the procedures, with the help of which it is possible to compute the unknown values and the spectra for the nonmonoenergetic composition of initial flow.

§ 1. Integral characteristics of the passing electrons.

Integral characteristics are important for estimation of efficiency of protection from electrons, for calculations of output of secondary particles from protection, etc.

Coefficients of transmission according to number of particles and on energy T_n are functions of energy of electron, atomic number, thickness of the layer and angle of incidence and are determined from relationships/ratios

$$T_N = \frac{\sum_i n(x, E_i) \Delta E_i}{N_0}, \quad (4.1)$$

$$T_E = \frac{\sum_i E_i n(x, E_i) \Delta E_i}{E_0 N_0}, \quad (4.2)$$

where $n(x, E_i) \Delta E_i$ - number of electrons, which emerge from layer of substance, with a thickness of x g/sm² in energy range $E_i \div E_i + \Delta E_i$;

N_0 and E_0 - number of falling/incident to layer electrons and their energy.

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In order to consider statistical error during calculation of coefficients T_N and T_E (R_N and R_E), we used conventional value of standard deviation ¹

$$\sigma = \bar{q} \sqrt{\frac{D[q]}{N_0 (\bar{q})^2}}, \quad (4.3)$$

where \bar{q} - mathematical expectation of random variable q ;

N_0 - number of trajectories, $D[q] = \bar{q}^2 - (\bar{q})^2$.

FOOTNOTE ¹. Less commonly for the evaluation/estimate is used the value of the probable (middle) deviation $E=0.67456\sigma$. ENDFOOTNOTE.

Dispersion of value q depends on two forms of errors: fluctuations in number of particles, which intersect this boundary, and fluctuations of radiation characteristics Q (energy, direction). Then, according to Berger [60],

$$\frac{1}{q} = \sqrt{\frac{1-p}{pN_0} \cdot \frac{Q^2 - (\bar{Q})^2}{(\bar{Q})^4}}, \quad (4.4)$$

where p - penetration probability.

In the case, when we are interested in the error in coefficient of transmission according to the number of particles, $\bar{q} = p \cdot T_N \cdot Q - 1$ and

$$\sigma = \sqrt{\frac{T_N(1 - T_N)}{N_0}}. \quad (4.5)$$

Standard deviation during the calculation of the coefficient of transmission according to the energy is computed from the formula

$$\sigma = T_E \sqrt{\frac{1 - T_N}{T_N N_0} \cdot \frac{\bar{E}^2 - (\bar{E})^2}{(\bar{E})^4}}. \quad (4.6)$$

Dependence $T_N(x)$ and $T_E(x)$ is a smooth curve, which collapses with height x . If we isolate on this the curved straight portion (i.e. the region, where $\frac{dT_N}{dx} = \text{const}$) and to continue it before the intersection with axis x (see Fig. 22), then the corresponding depth of penetration was called extrapolation length r_{N_0} .

As SELIGER [96] showed, it is convenient to express depth of penetration of electrons in units of complete path R_0 ².

FOOTNOTE ². It should be noted that many authors do not frequently give values of R_0 , as a result of which the uncertainty/indeterminacy upon transfer to the true thicknesses appears. ENDFOOTNOTE.

In this case, at least, for the energies ≤ 1 MeV the experimental curves of transmission for the different energies virtually are combined. The same conclusion follows also from our calculations [89]. Subsequently it was proposed to express depth of penetration in the units of extrapolation length. Since landing run r_e - important characteristic, you give the empirical formula for the extrapolation length, obtained during the interpolation designed r_N for elements C, Al, Si, Ti, Ge, Sn, Pb in the region of wave energies from 0.4 to 6.0 MeV and angles of incidence from 0 to 81° :

$$r_{N_0}(E, Z, \psi_0) = \cos^2 \frac{\psi_0}{2} \left[\frac{107.2 - Z}{5.442Z - 1312.0} + \frac{292.7 - Z}{4.163Z + 561.3} E + \right. \\ \left. + \frac{Z - 2.797}{63.86Z + 587.5} E^2 \right] \cdot 2 \text{ cm}^2, \quad (1) \quad (4.7)$$

where ψ_0 - angle of incidence;

E - energy of electron in MeV;

Z - the atomic number of element.

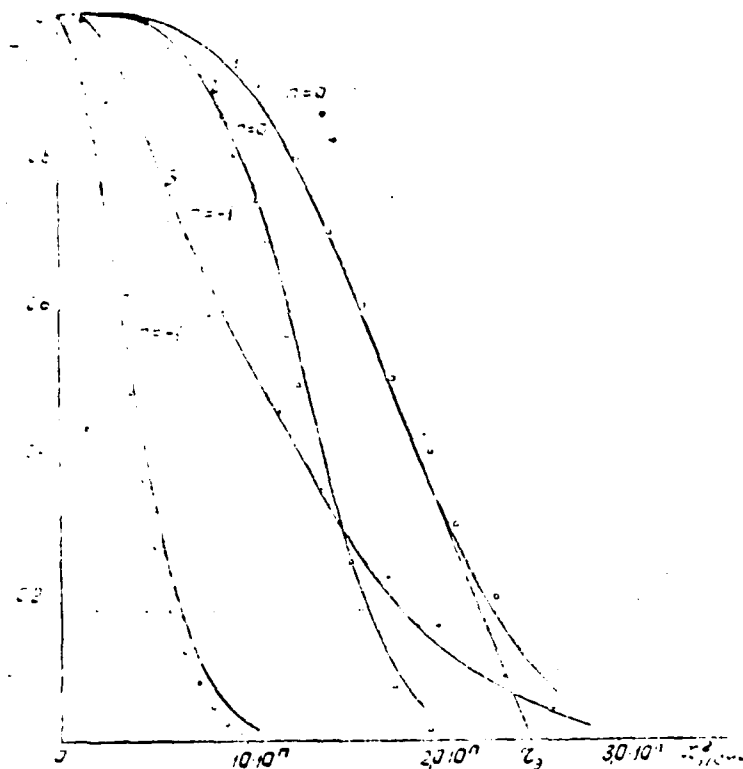


Fig. 22. Dependence $T_N(x)$ for perpendicular incidence/drop in electrons. Unbroken curves - the results of calculation, point - experiment. 1 - Cu, $E_0=6.0$ MeV; 2 - Al, $E_0=4.0$ MeV; 3 - Pb, $E_0=1.0$ MeV; 4 - Sn, $E_0=0.4$ MeV; n - scale index.

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By analogy with extrapolation length, calculated from their curves of transmission $T_N(x)$, the extrapolation length from curves of transmission is designed according to energy. Empirical formula for r_{E_0} takes the form

$$r_{E_0}(E, Z, \psi_0) = \cos^2 \frac{\psi_0}{2} \left[\frac{Z-187.0}{2344.0-7.625Z} + \frac{1400.0-Z}{37.82Z+3708.0} E + \frac{Z+14.14}{80.0Z+2207.0} E^2 \right]. \quad (4.8)$$

The values of extrapolation length obtained by us will satisfactorily agree with the experimental data of Ebert, etc. [97], Kononov [98], Radziyevskiy and Osanov [99], moreover it is better with the results of the first work.

From the data of Table 12 it is evident that calculated by us values r_N systematically less than designed by Baranov, etc. [100]. Apparently, this difference is caused by the fact that in the latter/last work the fluctuations in energy losses to the ionization and the bremsstrahlung are not considered. For describing the empirical dependence $T_N(x)$ and $T_E(x)$ was used the formula.

$$T_{N,E} = \exp \left[-\beta \left(\frac{x}{r_s} \right)^\alpha \right], \quad (4.9)$$

which is for the first time proposed in the work of Makhov [101]. The same late expression was used by Mar [88]. We obtained the following empirical relationships/ratios for the coefficients α and β by working/treatment according to the method of least squares of the curves of transmission. For T_N

$$\alpha = \begin{cases} 1 + \frac{5,6-0,1(3,4-E)^2}{Z^{0,348-0,082E}} (u-0,1564) + \\ \quad + 0,0125 (E-2) (50-Z) \times \\ \quad \times (u-0,1564)^2, \text{ для } Z < 50, E > 2 \\ 1 + \frac{5,5-0,1(3,4-E)^2}{Z^{0,358-0,082E}} (u-0,1564) \text{ в остальных} \\ \quad \text{случаях,} \end{cases} \quad (4.10)$$

Key: (1). for. (2). in ^{the} rest of (3). ^{the} cases.

where $u = \cos \psi_0$, E в Мэв, $\cos 81^\circ = 0,1564$.

$$\beta = 2,59 - 0,0076 (Z - 6), \quad Z > 6. \quad (4.11)$$

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For T_E

$$\alpha = 0,708 + \frac{Z+24}{0,93Z+13,7} (u - 0,1564), \quad (4.12)$$

$$\beta = \frac{Z+32,6}{0,524Z+10,8}. \quad (4.13)$$

Calculated according to empirical formulas (4.9-4.13) curves of transmission $T_N(x)$ and $T_E(x)$ will be agreed on with experiment (Fig. 22) with accuracy, by best 10% for angles of $\psi_0 < 45^\circ$, and depths, for which $T_{N,E}(x) > 0,1$.

Table 12. Comparison of the calculated and experimental values of extrapolation length $r_{N, \text{ссм}^2}$.

		(1)											
E		6 Mэв				4 Mэв				2 Mэв			
Z		экспе- римент (97) (2)	экспе- римент (98) (2)	наши данные (3)	экспе- римент (97) (2)	экспе- римент (98) (2)	наши данные (3)	экспе- римент (97) (2)	экспе- римент (98) (2)	наши данные (3)	экспе- римент (98) (2)	наши данные (3)	расчет [100] (4)
3,6		—	—	2,803	—	—	1,882	—	—	0,884	—	—	1,129
6		3,150	3,192	2,974	1,953	2,097	1,930	1,002	0,912	—	—	—	1,020
13		2,967	3,080	2,910	1,837	2,020	1,857	0,960	0,832	—	—	—	0,874
14		2,942	3,064	2,897	1,821	2,009	1,845	0,954	0,855	—	—	—	0,861
22		2,780	2,936	2,762	1,706	1,921	1,740	0,906	0,777	—	—	—	0,786
32		2,561	2,776	2,583	1,580	1,811	1,613	0,818	0,729	—	—	—	0,730
50		2,267	2,448	2,287	1,395	1,613	1,407	0,738	0,626	—	—	—	0,687
82		1,881	1,976	1,881	1,153	1,261	1,122	0,546	0,492	—	—	—	0,605

E		1 Mэв				0,7 Mэв				0,4 Mэв			
Z		экспе- римент (98) (2)	наши данные (3)	расчет [100] (4)	экспе- римент (99) (2)	наши данные (3)	расчет [100] (4)	экспе- римент (98) (2)	наши данные (3)	расчет [100] (4)	экспе- римент (98) (2)	наши данные (3)	расчет [100] (4)
3,6		0,388 ¹	0,396	0,512	0,260	0,245	0,330	0,127	0,114	—	—	—	0,155
6		0,454	0,413	0,483	—	0,265	0,299	—	0,117	—	—	—	0,141
13		0,430	0,386	0,396	—	0,246	0,256	—	0,107	—	—	—	0,120
14		0,426	0,382	0,391	—	0,243	0,252	—	0,106	—	—	—	0,119
22		0,398	0,353	0,357	—	0,223	0,230	—	0,0959	—	—	—	0,108
32		0,364	0,320	0,331	—	0,202	0,214	—	0,0858	—	—	—	0,101
50		0,300	0,273	0,303	—	0,172	0,195	—	0,0732	—	—	—	0,0920
82		0,188	0,218	0,274	—	0,141	0,177	—	0,0664	—	—	—	0,0833

Key: (1). (normal incidence). (2). experiment. (3). our data.
(4). calculation.

FOOTNOTE ¹. The value indicated is undertaken by IS of work [99].

ENDFOOTNOTE.

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For large angles and large depths approximation (4.9) gives error 20-30%. Formulas (4.9-4.13) describe the coefficients of transmission only for $Z \geq 6$. For the light substances, for example Plexiglas, we give the appropriate values of these coefficients in the Table 9

Appendices.

§ 2. Integral characteristics of the back-scattered electrons.

Backscattering of electrons is of interest for many branches of nuclear physics and technology. In this direction many experimental works are carried out. The first data on back scattering by the Monte Carlo method are acquired in the work of Berger [102]. As in the case of passage, they are frequently interested in the integral coefficients of reflection R_N and R_E , which are defined as analogous to T_N and T_E .

We will here speak only about complete coefficients of reflection R_N and R_E , i.e. about coefficients, which correspond to reflection of electrons from semi-infinite barrier.

Curves $R_{N,E}$ monotonically increase from 0 to value, which corresponds to complete reflection coefficient. Saturation is achieved actually at depths $x \sim 0.3 R_0$. We did not attempt to describe designed [92] data for R_N and R_E by empirical formula. These data in the form of graphs for different angles of incidence and wave energy of electrons are shown in figures 23 and 24, in which points gave the averaged experimental results, obtained for the perpendicular incidence/drop in the electrons.

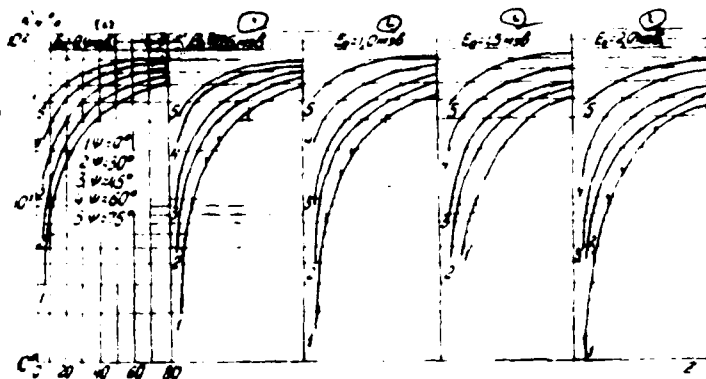


Fig. 23. Countesses of dependence $R_N(Z)$ for different energies and angles of incidence ψ .

Key: (1). MeV.

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The statistical accuracy of the results of calculation was not less than 10%, to the decrease of the fluctuations of results contributed the methodology of "splitting/fission" and "tape measures". It is possible to consider that the results of calculation will completely satisfactorily be coordinated with the experiment.

Special feature of electronic trajectories is such, that an electron, which has turned back at small depth, as a result of "successful" selection can leave sample. The probability of scattering to the large angle is small and therefore the results of calculation will be sensitive both to the form of the section in this region of angles and, possibly, to the selected diagram of Monte Carlo. In Table 13 we are congruent/equate the results of our calculations $R_{N,E}$ with the results of calculating Berger [102].

From the data of Table 13 it is evident that results of both calculations satisfactorily will be coordinated with each other, although they are carried out with the help of different diagrams of Monte Carlo. Meanwhile the attempts to spread these calculations to the high-energy region lead to essential divergence R_N from experiment [130], as is evident from the data of Table 14.

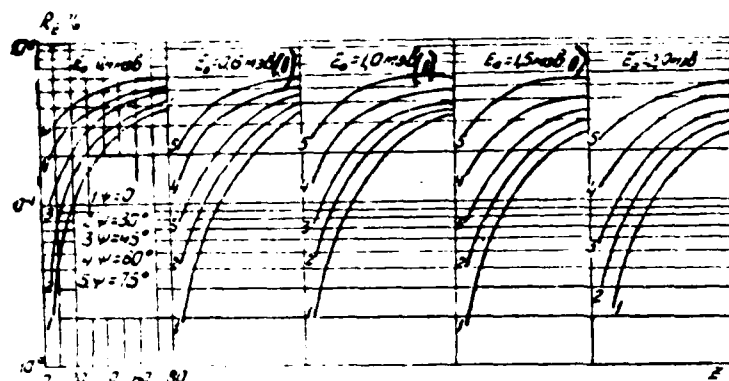


Fig. 24. Graph/diagrams of dependence $R_E(Z)$ for different energies and angles of incidence ψ .

Key: (1). MeV.

Table 13. Values R_N, R_E . (%), $E_0 = 1$ MeV, $Z = 13$.

ψ_0	0	35°	45°	60°	75°	90°
[102] R_N маши данные (1)	9,0	—	22,0	33,0	49,4	79,6
[102] R_E маши данные (1)	7,1 ± 0,8 4,4	13,4 ± 1,1 —	18,8 ± 1,2 12,8	34,6 ± 1,2 20,8	50,9 ± 1,6 36,5	— 72,5
	4,0	7,1	9,2	20,3	37,2	—

Key: (1). our data.

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Partially this disagreement can be eliminated by selecting new reduced coefficients of k (see Table 8) in order to increase "wandering" of electron in near-surface layers. Conducting further calculations and electrons, refined sections of Mott for the large scattering angles, etc is required.

In Table 15 are given values of medium energies \bar{E} of back-scattered electrons for different angles of incidence.

Table 14. Values R_N %, $E_0 = 4$ MeV, $\psi_0 = 0^\circ$.

C		Al		Cu		Au(Pb)	
эксперимент	расчет	эксперимент	расчет	эксперимент	расчет	эксперимент	расчет
0,70 ± 0,09	0,25 ± 0,03	3,2 ± 0,4	1,68 ± 0,17	10,2 ± 1,2	4,6 ± 0,8	27,4 ± 1,9	10,9 ± 1,6

Key: (1). experiment. (2). calculation.

Table 15. Medium energy E of the back-scattered electrons.

E_0, MeV	0°	30°	45°	60°	75°
$Z=6$					
0,4	0,118	0,192	0,182	0,218	0,277
0,6	0,244	0,282	0,287	0,321	0,394
1,0	0,362	0,459	0,480	0,533	0,636
1,5	0,364	0,428	0,517	0,661	0,932
2,0	0,588	0,528	0,987	0,907	1,285
$Z=13$					
0,4	0,211	0,229	0,244	0,264	0,277
0,6	0,183	0,296	0,338	0,389	0,432
1,0	0,416	0,511	0,542	0,587	0,731
1,5	0,530	0,621	0,724	0,806	1,019
2,0	0,738	0,812	0,917	1,061	1,299
$Z=29$					
0,4	0,239	0,233	0,286	0,303	0,330
0,6	0,256	0,347	0,419	0,445	0,477
1,0	0,551	0,528	0,652	0,735	0,806
1,5	0,738	0,694	0,905	0,997	1,112
2,0	1,028	1,042	1,155	1,294	1,526
$Z=50$					
0,4	0,293	0,268	0,315	0,327	0,341
0,6	0,411	0,426	0,449	0,469	0,503
1,0	0,687	0,701	0,728	0,754	0,826
1,5	0,835	0,859	1,033	1,065	1,169
2,0	1,167	1,154	1,364	1,392	1,523
$Z=82$					
0,4	0,316	0,309	0,318	0,341	0,347
0,6	0,467	0,477	0,486	0,500	0,521
1,0	0,726	0,739	0,757	0,790	0,852
1,5	1,018	0,990	1,086	1,117	1,208
2,0	1,258	1,296	1,341	1,448	1,607

Key: (1). MeV.

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As can be seen from Table 15, \bar{E} monotonically they grow with increase of Z and ψ_0 only sometimes are small divergences from monotonicities, caused by statistical straggling.

For obtaining reliable data about angular and energy distributions of electrons reflected is required too long a time, even if we use methods of accelerating calculation.

§ 3. Energy and angular spectra of the electrons, which passed through plane layer.

Energy and angular spectra of electrons, which passed layer of substance, contain much greater information than integral characteristics. The general view of the energy spectra of electrons, passing through the consecutive layers Ge, it is shown in Figure 25. Their form varies from the curve almost gaussian with the small thicknesses to the wide of asymmetric form with the large thicknesses. Unfortunately, we did not succeed in fitting the empirical formula, which describes these spectra at any depths, with a sufficient accuracy.

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The formula of the form

$$n(E)dE = CE^\alpha(E_m - E)^\beta dE, \quad (4.14)$$

where E_m - maximum energy of electrons in the spectrum, and α , β and C

are functions Z , E_0 , ψ_0 , it can be used only on the large depths, but not for any thicknesses, as this is indicated in work [103].

As sufficiently rough approximation of spectra of fogs [88] it proposed expression

$$n(E, E_0, Z, x) dE = \begin{cases} A \exp[-b(E_0 - E)] dE & \text{при } E \leq E_m, \\ 0 & \text{при } E > E_m. \end{cases} \quad (4.15)$$

Key: (1). with.

Parameters b and A are determined from the relationships/ratios

$$b = \left(\frac{x}{E} \right)^{-1.46} (1.53 - 0.0147 Z) / E_0, \quad A = \frac{b T_N \exp(b E_0)}{\exp(b E_m) - 1},$$

where T_N - coefficient of transmission.

Our results are represented in the form of matrix/die of values for density of distribution $n(E)$ (μ/Mev), designed for several thicknesses, maximum from which correspond $T_N(x_m) \approx 0.1$, and 10 values of energies E_n on energy scale $\frac{E}{E_m}$, where E_m (MeV) wall energy in spectrum, and

$$E_n = \frac{E_m}{20} (2n+1), \quad n = 9, 8, 7, \dots, 1, 0,$$

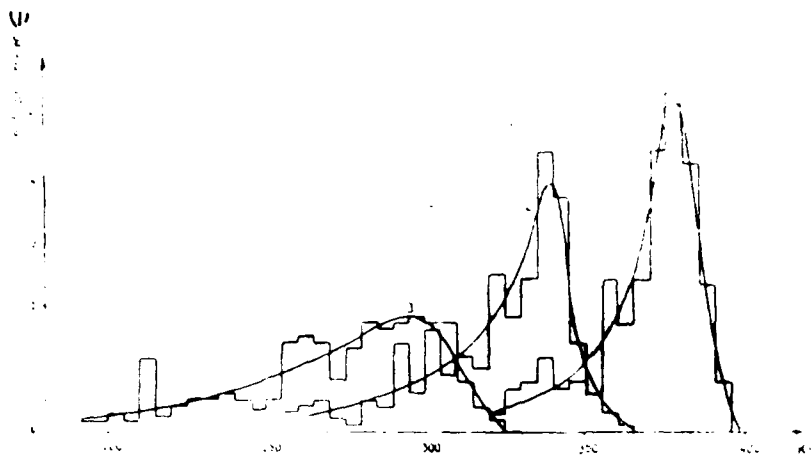


Fig. 25. Energy spectra of electrons $E_0=1.5$ MeV after passage of layers Ge of different thickness: 1 - 80.7 mg/cm^2 ; 2 - 106.0 mg/cm^2 ; 3 - 129.4 mg/cm^2 .

Key: (1). rel. un.

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Values $n(E_n, x)$ for different substances, wave energies and angles of incidence of electrons together with wall energies E and thickness of the layer $x \text{ g/cm}^2$ are given in the Table 10 applications/appendices. All spectra are calibrated to one. For the production of a spectrum taking into account the falling/incident electron values $n(E_n)$ from this matrix row must be multiplied by $T_N(x)$. It is natural that the accuracy of reproduction of the true spectra with the help of the tables is low for the small thicknesses (1-2 table rows), since in this case in one interval $\frac{E_n}{10}$ almost entire spectrum is included. However, the following lines of matrix/die describe the spectrum with the satisfactory accuracy.

In Figure 26 calculated energy spectrum is equal with spectrum, obtained under the same conditions on beta-ray spectrometer by Andreev [104]. Agreement is very good. A good agreement is obtained also during the comparison of calculation from experiments [105] for Si ($E_0=0.4$ MeV). The majorities of the authors, who investigated the spectral composition of the passing through plane layer electrons, usually give these characteristics: the probable losses of energy $\Delta E = E_0 - E_1$ and average/mean losses $\overline{\Delta E} = E_0 - \overline{E}$, where E_1 - the most probable energy in the spectrum, and \overline{E} - medium energy of the passing electrons.

Table 16 gives these characteristics, obtained from experiments of Kononov et al. [106] and from our calculations.

Small systematic disagreement between calculated and experimental values is caused by differences in energies, for which are carried out calculations ($E_0=6.0$ MeV) and experiment ($E_0=6.7$ MeV).

Relation $k = \frac{\overline{\Delta E}}{\Delta E_1}$, according to data of work [106], weakly increases from $k=1.15$ ($Z=6$) to $k=1.60$ with ($Z=83$).

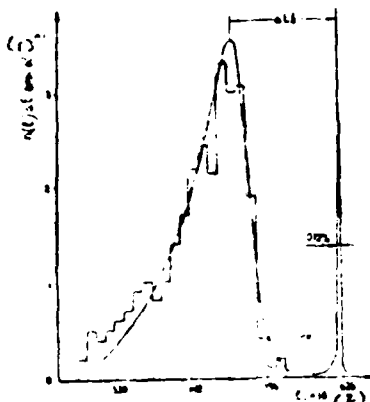


Fig. 26. Comparison of experimental and calculated spectra of electrons ($E_0 = 0.624$ MeV) after passage of layer Au with thickness of 38.35 mg/cm^2 . The instrument width of line is to the right shown, ΔE - probable losses of energy.

Key: (1). rel. un. (2). keV.

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✓ According to our data, for the energy $E_0 = 6.0$ MeV $k = 1.20 \pm 0.10$. In the work of Baranov, etc. [107] ($E_0 \leq 3$ MeV) $k = 1.18 \pm 0.24$ does not depend on Z and x . Our results in this range of energies do not confirm this conclusion, since, first, relation $\frac{\Delta E}{\Delta E_0}$ is function x and has a maximum for $x/R_0 = 0.15 - 0.03$, and values k in the maximum vary from 1.20 for carbon to 1.80 for tin; secondly, with an increase in thickness $k \rightarrow 1$. This disagreement is the corollary of disregard into [107] fluctuations in energy losses.

The data, given in Table 10 Appendices, can be used also for determining average/mean from energy spectrum of electrons.

Calculated by Monte Carlo method angular distributions completely satisfactorily will be agreed on with experiment [108, 109]. In figure 27 is carried out the comparison of calculation with the experimental data of Lonergan et al [109] for Al and Au with the energy of the falling/incident electrons $E_0=4$ MeV.

Angular distribution with small thicknesses, as show experiment and our calculation, is satisfactorily described by Moliere's curve, if we introduce small corrections as this recommended in work [108]. For the large depths the distribution of electrons approaches the diffuse:

$$f(\theta) \sim \cos^2 \theta.$$

Table 16. Comparison $\Delta E_{\text{эксп}}$ and $\Delta E_{\text{расч}}$

C				Al				Cu(Ge) ¹			
$x, \text{ g/cm}^2$	1,78	1,25	0,53	1,89	1,35	0,54	0,27	1,34	0,90	0,45	0,27
$\Delta E_{\text{эксп}}, \text{ МэВ (1)}$	3,50	2,45	1,07	3,85	2,65	1,12	0,53	2,82	1,72	0,86	0,55
$\Delta E_{\text{расч}}, \text{ МэВ (1)}$	3,65	2,50	1,10	3,85	2,75	1,15	0,55	2,80	1,75	0,88	0,50

				Sn				Pb			
$x, \text{ g/cm}^2$	1,08	0,83	0,51	0,24	1,13	0,87	0,60	0,25			
$\Delta E_{\text{эксп}}, \text{ МэВ (1)}$	2,40	1,75	1,04	0,48	2,77	2,08	1,40	0,54			
$\Delta E_{\text{расч}}, \text{ МэВ (1)}$	2,25	1,75	1,10	0,45	2,85	2,20	1,50	0,65			

Key: (1). MeV.

experimental values - for Cu (Z=29)

FOOTNOTE ¹. Computed values are obtained for Ge (Z=32). For Cu(E₀=2.65 MeV) we obtained a good agreement of probable loss with the experimental data of work [48]. ENDFOOTNOTE.

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Obtained in calculation angular distributions for $x/R_0 > 0.4$ are described well by this distribution (unbroken curve in Fig. 27)

$$f(\theta) \sin \theta d\theta = 3 \cos^2 \theta \sin \theta d\theta \quad (4.16)$$

(function $f(\theta)$ it is calibrated to one). In this case the mean angle of scattering proves to be constant and equal to

$$\begin{aligned} \theta &= \int_0^{\frac{\pi}{2}} \theta f(\theta) \sin \theta d\theta = \\ &= \frac{2}{3}. \quad (4.17) \end{aligned}$$

Actually, the results of our calculation give value of $\theta = 35^\circ - 40^\circ$ for the mean angle independent of the wave energy of electron, atomic number and angle of incidence. However, to a similar result comes Baranov [107]. Other by Berger proposed expressions for the function distribution

$$f(\theta) = \begin{cases} 2 \cos \theta \\ 1.445 (0.717 \cos \theta) \\ 1 - \cos^2 \theta \end{cases} \quad (4.18)$$

it leads to an increase in mean angle of θ from 35 to 42°, that, obviously, it is not substantial with the evaluation of not very important in the problems of transfer characteristic.

§ 4. Distribution of absorbed energy.

Beams of accelerated electrons began in recent years to play large role in national economy. With their aid ultrapure metals and

alloys are obtained; they extensively are used for the welding and in radiochemical technology.

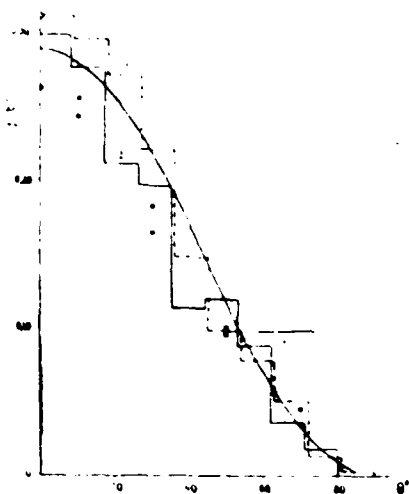


Fig. 27. Angular distribution of electrons ($E_0=4$ MeV), which passed layers: $x=1.275$ g/cm² Al (histogram 1 - calculation, light small circles - experiment); $x=1.460$ g/cm² Au (histogram 2 - calculation, dark/nonluminous small circles - experiment). Continuous curve - $f(\theta)=3 \cos^2 \theta$.

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Finally, one additional very important application - radiobiology. For the practical applications/appendices it is important to know the spatial distribution of absorbed energy, the appearing as corollary of two processes - energy losses to the ionization and the excitation of electron shell and multiple scattering of electrons in the substance. The distribution curves of absorbed energy at the angles of incidence of electrons of $\theta \leq 45^\circ$ have characteristic bell-shaped form with the maximum, which lies on depth $x \sim 0.3 R_0$. The appearance of this maximum is caused by the contribution to the energy, absorbed in this layer

both from the passing electron stream, and from the flow, reflected of the deeper layers. Obtained in the experiment of the spatial distributions of absorbed energy - difficult problem. Difficulties are connected with the inadequacy of the methodology of the determination of absorbed energy with the help of ionization chambers, especially for the substances with large Z . Therefore at present the majority of experimental data is obtained for light elements [110] and for the so-called "tissue-equivalent" substances. For the heavy elements the most precise measurements of the distributed absorbed energy were made by Nakai [111]. Is recently developed new methodology [112], which over the long term will make it possible to obtain the distributions of absorbed energy with the satisfactory accuracy for any substances. But thus far calculated methods remain fundamental for obtaining these distributions.

Processing our calculation data made it possible to obtain empirical expression for distribution in depth $\frac{x}{r_{N_0}}$ of absorbed energy

$$\frac{\Delta E(x)}{\Delta x} = A \left[1 + b \left(\frac{x}{r_{N_0}} \right)^a \right] T_N \left(\frac{x}{r_{N_0}} \right), \quad \frac{M_{30} \cdot \text{cm}^2}{z}. \quad (4.19)$$

Here $\Delta E(x)$ - energy, absorbed in layer Δx at depth x in the sample; T_N - coefficient of transmission, computed from formula (4.19).

Constants A , a , b are functions E , Z , u for the elements $14 \leq Z \leq 50$ are located with the help of the formulas

$$A = \frac{E+2.14}{0.45+0.60E} + 5.17(1-u); \quad b = [1.10(2-u) - 0.43]^{-2};$$

$$a = 2.15 \cdot \frac{[1.21 + (1-u)](Z-14)}{38.78 + 392.14(1-u)},$$

where E - energy of electron in MeV, $u = \cos |\psi_0|$. The accuracy of the description of calculated distribution by formula (4.19) composes \pm

20%) for angles of $\psi_0 < 45^\circ$ and about 30% for angles of $45^\circ \leq \psi_0 < 63^\circ$.

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The agreement of calculated distributions $\Delta E(x)/\Delta x$ for Al and Pb ($E_0 = 1$ MeV) with experimental results of [111], as can be seen from figure 28, ^{is} very good. For Plexiglas, important simulating connection in the radiobiology, we computed the distribution of absorbed energy [113]. They are given in Figures 29-34. Transition to the absorbed dose (caused by flow $\Phi = 1 \text{ cm}^{-2}$) is realized the multiplication of absorbed energy to the constant factor $1.6 \cdot 10^{-8} \text{ rad/MeV}$.

Knowing values of absorbed energy on this depth, it is possible to compute such important characteristics as value of the induced charge in dielectrics and quantity of surplus carriers (electrons and holes) in semiconductor.

Recently Kobetich and Katz [114] proposed an empirical method for determination of distribution $\Delta E(x)/\Delta x$.

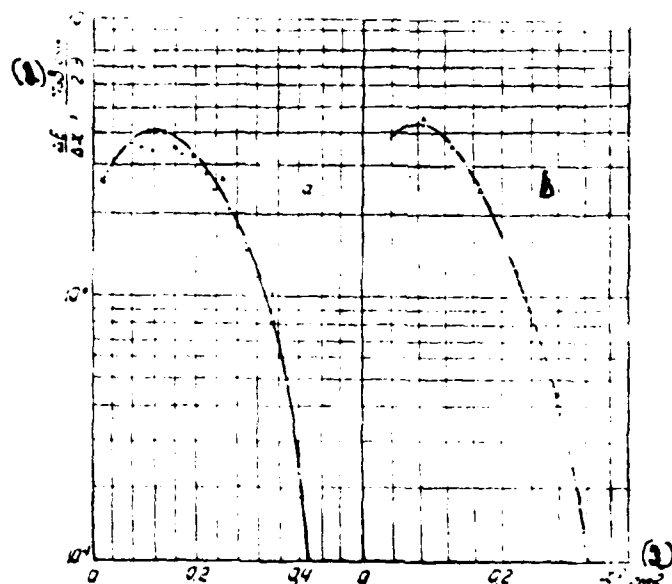


Fig. 28. Distribution of absorbed energy in depth. a - in Al; b - in Pb. $E_0=1.0$ MeV. Unbroken curves - calculation, points - experiment. Key: (1). MeV cm². (2). g/cm².

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Calculations according to this method are connected with the numerical differentiation of a certain function, which is the product of the coefficient of transmission T_N and polynomial, which expresses the connection/communication of residual range with the energy. Although obtained thus distribution will agree well with the experiment, it is unclear, as it is possible to spread this method to the case of oblique incidence in the electrons.

§ 5. Application of the obtained results.

Results, presented in previous paragraphs, can be used for solving wide circle of practical problems. Let us name some of them here only.

1. Determination of characteristics of passing electron stream for polyatomic systems, in particular organic. Calculations showed that in this case it is possible to use the systems described above of Monte Carlo for a certain hypothetical monoatomic substance with the effective values of atomic number Z_{eff} , of atomic weight A_{eff} and ionization potential I_{eff} .

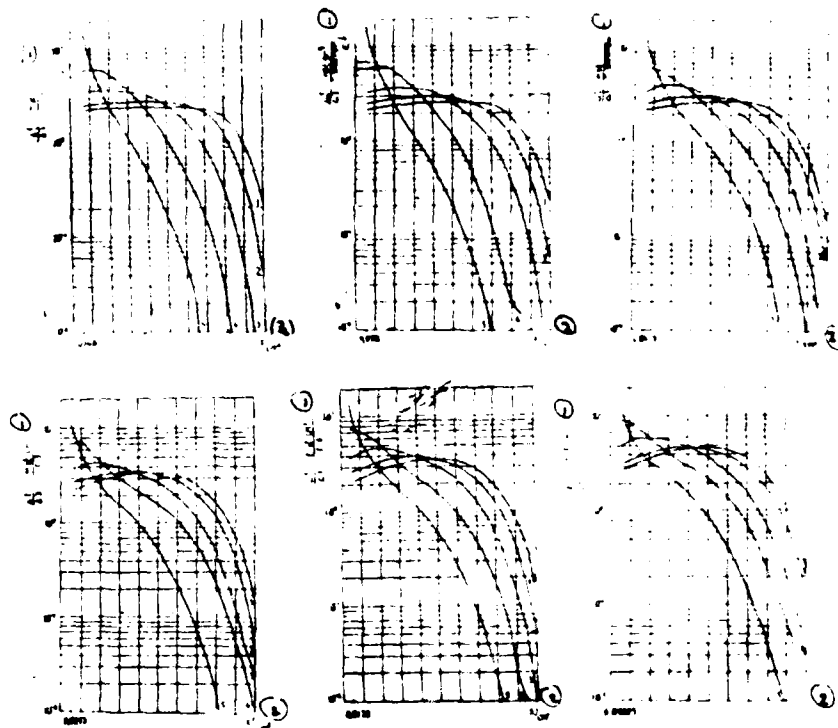


Fig. 29-34. Distribution in depth of absorbed energy in Plexiglas for different wave energies $E_0 = 6, 4, 2, 1, 0.7, 0.4$ MeV, respectively and angles of incidence: 1 - $\psi_0 = 0^\circ$; 2 - $\psi_0 = 27^\circ$; 3 - $\psi_0 = 45^\circ$; 4 - $\psi_0 = 63^\circ$; 5 - $\psi_0 = 81^\circ$.

Key: (1). MeV cm. (2). cm^2 .

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These values are computed from the formula:

$$M = \sum_i \rho_i M_i, \quad (4.20)$$

where $M = A, Z, I$; M_i — their partial values;

ρ_i — the concentration by weight of this component in the system.

From Table 17, where are given effective values Z , A , I for series/row of organic matter and water, it is evident that they differ little from each other (with exception of polyethylene). Therefore data of calculations for Plexiglas can be used also for the series/row of other substances, in particular for the animal cloth.

Calculations show that coefficients of transmission, found thus for NaI ($Z_{\text{eff}} = 32$), also well will agree with experiment.

2. Determination of coefficients T_N and T_E for arbitrary energy and angular distribution of electrons $2\pi f(E, u) dE du$. It is clear that in this case

$$T_{N,E} = 2\pi \int_{E_{\min}}^{E_{\max}} \int_0^1 T_{N,E}(u, E, Z, x) f(E, u) dE du, \quad (4.21)$$

where $u = \cos \psi$; E_{\min} and E_{\max} - wall energies in the spectrum. We in work [115] have carried out calculations $T_{N,E}$ for the spectrum of the electrons of Earth radiation belt.

3. Determination of approximate number of secondary electrons, dislodged/chased from barrier by thickness x g/cm² into front/leading hemisphere: a) by γ -radiation and b) by fast electrons during perpendicular incidence/drop.

a. Thickness of sample is assumed to be equal to landing run of photoelectron with energy $E = h\nu - I$ (I - electron-binding energy in K-layer), since with this thickness flow of secondary electrons achieves saturation.

Table 17. Effective values Z, A, I.

(1) Вещество	(2) Формула	Z _{эф}	A _{эф}	I _{эф} , %
Плексиглас(3)	C ₅ H ₈ O ₂	3,6	6,67	45,8
Полистирол(4)	C ₈ H ₈	3,35	6,02	44,5
Полиэтилен(5)	CH ₂	2,67	4,67	36,3
Вода (6)	H ₂ O	3,3	6,0	40,0

Key: (1). Substance. (2). Formula. (3). Plexiglas. (4). Polystyrene. (5). Polyethylene. (6). Water.

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The output of secondary radiation was studied in work [116], and for n is obtained the following expression:

$$n = \frac{1}{2} \frac{N}{A} \int_0^x \int_0^1 e^{-\mu x} \left\{ Z \frac{d\sigma_k}{d\Omega} T_N[E_k(\psi), Z, x, u] + \frac{d\sigma_{ph}}{d\Omega} T_N[E_{ph}(\psi), Z, x, u] \right\} du dx, \quad (4.22)$$

where A - atomic of element;

μ - coefficient of absorption $\mu = \frac{N}{A} (\tau_k Z + \tau_{ph})$;

x - thickness in g/cm²;

$\frac{d\sigma_k}{d\Omega}$ the differential cross section of the Compton effect, determined by Klein-Nishina-Tamm formula;

$\frac{d\sigma_{ph}}{d\Omega}$ the differential cross section of photoelectric emission [117];

$E_{\alpha}(\psi)$, $E_{\phi}(\psi)$ - corresponding energies of a Compton-electron, scattered at angle ψ , and the photoelectron, dislodged/chased in the direction ψ ;

T_{α} coefficient of transmission.

In formula (4.22) integrated on azimuth angle is already carried out. The calculations of the output of secondary electrons for the thickness, which corresponds to medium energy Compton-and photoelectrons taking into account the static weight of these processes, are carried out in work [118]. In this case the thickness of sample will be somewhat smaller than the landing run of photoelectron. Entire process, beginning from the generation of secondary electrons and ending with their tracking in the sample, is simulated according to the Monte Carlo method.

Comparison of results of calculation according to formula (4.22) with our results is given in Table 18.

If we take into account aforesaid above about different thicknesses of sample in both calculations, then data of table can be considered as confirmation of method of analytical calculation of output of secondary electrons.

b. By analogy with γ -quanta it is possible to calculate quantity

of δ -electrons, which emerge from sample under action of fast electron (calculation method is schematically shown in figure 35).

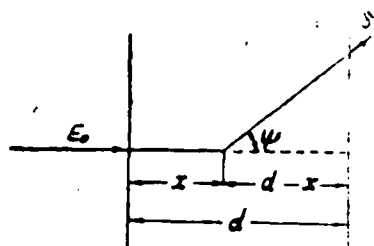


Fig. 35. Diagrammatic representation of "birth" of δ -electron at depth x , g/cm² in sample with thickness of d , g/cm².

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$$n_{\delta} = \frac{NZ}{2A} \int_0^{u_{\min}} \int_0^d T_N(E, Z, x) \frac{d[\bar{E}(x)]}{d\Omega} T_N[E(\psi), Z, d-x, u] du dx, \quad (4.23)$$

where $\frac{d[\bar{E}(x)]}{d\Omega}$ - section of Moller, determined by formula (1.23) for the medium energy of electrons in spectrum $\bar{E}(x)$;

$E(\psi)$ - energy of a δ -electron, which moves at angle ψ to the direction of motion of the initial electron (it is determined from formula 1.22);

$$u_{\min} = \left[\frac{E+1.02}{E+2.04} \right]^{1/2},$$

and E - energy in MeV.

Some results of calculations according to formula (4.23) are given in work [97]. It is shown that the noticeable contribution of δ -electrons to the coefficient of transmission can be expected only with the small thicknesses of sample and for the light elements.

4. Distributions for absorbed energy can be used for research of processes, in which participate fast polyvalent ions.

Table 18. Output of secondary electrons to one that falling of γ -quanta.

	Al					
$E, \text{MeV} \text{ (1)}$	2,0	1,5	1,25			0,7
$x, \text{г см}^{-2} \text{ (2)}$	0,564	0,384	0,034	0,282	0,614	0,095
$n, \times 10^3$	10,6	7,1	1,55 ^{b)}	5,5	7,1 ^{b)}	2,7

	Cu					
$E, \text{MeV} \text{ (1)}$	2,0	1,5	1,25			0,7
$x, \text{г см}^{-2} \text{ (2)}$	0,700	0,450	0,046	0,334	0,467	0,114
$n, \times 10^3$	7,8	7,3	1,95 ^{b)}	4,6	5,53 ^{d)}	2,0

Key: (1). MeV. (2). g/cm².

FOOTNOTE ¹. Data are undertaken from work [116]. ENDFOOTNOTE.

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In this case appears a large number of δ -electrons, as a rule short-run, and, consequently, the track of this ion is cylindrical tube with the very high density of absorbed energy.

Information about this energy is used for evaluation of these important characteristics: 1) light output from ionic crystals during recording of heavy ions; 2) sizes/dimensions of track in nuclear emulsion; 3) efficiency of track defects; 4) radiobiological effect from fission fragments, etc.

Work [114] gives formula for energy distribution of δ -electrons, formed on 1 cm of path by ion with effective charge $(Ze)_{\text{eff}}$

$$\frac{dn}{d\omega} = \frac{2\pi N e^2 (Ze)_{\text{eff}}^2}{m_0 c^2 \beta^2 \omega^2} \left[1 - \beta^2 \frac{\omega}{\omega_m} + \pi \alpha \beta \left(\frac{\omega}{\omega_m} \right)^2 \left(1 - \frac{\omega}{\omega_m} \right) \right], \quad (4.24)$$

where N - number free electrons in 1 cm³ of substance

$$(Ze)_{\text{eff}} = Ze [1 - \exp(-125 Z^{-1/2})]$$

and

$$\omega_m = \frac{2m_0 c^2 \beta^2}{(1 - \beta^2)^{3/2}}$$

Kinetic energy of electron E is determined on known energy of binding I [119] of the relationship/ratio $E = \omega - I$. From the kinematic considerations it follows that between the angle of emission ψ of a δ -electron with respect to the trajectory of ion and its energy there is a dependence $\cos \psi = \left(\frac{\omega}{\omega_m} \right)^{1/2}$. Since $\omega < \omega_m$, that $\psi > \frac{\pi}{2}$. In order to find the distribution of absorbed energy along the track of ion, it is necessary to compute the expression

$$\frac{dE}{dr} = \int_I^{E_m} \int_0^{t_m} \frac{dE}{dt} [\beta(r), E] \frac{d^2 n}{dE dr} [\beta(r)] dE dt, \quad (4.25)$$

where $dE/dr [\beta(r), E]$ - the distribution of the absorbed energy is determined with the help of the formula (4.19) or from the experiment, $d^2 n/dE dr$ it is obtained from (4.24) taking into account dependence $\beta = \beta(r)$. The latter is found from relationship/ratio a landing run-energy for the ion. Limit t_m in integral (4.25) corresponds to maximum energy $E_m = \omega_m - I$.

5. In connection with requirements of radiation physics of

semiconductors recently especially increased interest in research of radiation defects, formed in Si and Ge by fast electrons.

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This is explained the possibility of variation over wide limits of energy of electrons and introduction thus to the sample of the defects of different forms, and also fact that the electrons of not very high energy ($E_0 < 20$ MeV) virtually do not induce activity in the irradiated object and experiment is pure/clean. For the same reasons, which cause the distribution of absorbed energy in the sample, appears the spatial distribution of defects. Knowledge of this distribution is important for a whole series of applied questions of radiation physics and technology.

In work [89] we showed that it is easy to obtain him, combining results of calculations of spectra of electrons at given depth $n(x, E)$, obtained by Monte Carlo method, with experimental cross-section $\sigma(E)$ of formation/education of defects. A quantity of defects, formed in the sample at depth $x > x_1$ (it is examined the one-dimensional case), it is possible to compute according to the formula

$$N(x - x_1) = \int_{E_{min}}^{E_{max}} n(x, E) \sigma(E) dE, \quad (4.26)$$

where E_{min} corresponds to the threshold energy of the formation/education of defects (in Si, $E = 0.15$ MeV);

E_{max} maximum energy of electrons in the spectrum.

In order to find distribution itself, it is necessary to construct dependence

$$N(x) \sim \frac{d^+(x)}{dx}.$$

Figure 36 shows dependence on depth of probability of forming defects $\eta(x) \sim N(x)$ in Si with wave energy of electrons $E_e = 0.5$ MeV. In the formula experimental section from the work of Loferski, etc. [120] was used. The results of calculation (curve 1) and experiment (curve 2) satisfactorily will be coordinated between themselves. Curve 3 is designed Yurkov [121] with the help of the analytical method, similar to Spencer's method. The difference between both calculations should be related due to the impossibility in this stage of correct account in the equation of the transfer of fluctuations in the losses of energy and multiple scattering.

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Subsequent calculations [122] are carried out for the broader band of energies of electrons, moreover as the probability of forming the defects in n-silicon by monoenergetic electrons formula [123]

$$\eta(E) = 0,18 \ln 4E, \frac{\text{дефект}}{\text{эл. см}}, \quad (4.27)$$

where E - energy of electrons in MeV, is used. Threshold energy is taken in this case as the equal to 0.25 MeV. The results of calculations for different wave energies and normal incidence are shown in Figure 37. The case, when silicon is covered with the layer of carbon, is investigated also, i.e., defect formation occurs after

the passage of the layer of protection x g/cm².

For convenience thickness of sample Si or protection from carbon is expressed in units of complete landing run

$$R_{0\text{ Si}} = 0,1052 + \\ + 0,6784E - 0,0077E^2.$$

$$R_{0\text{ C}} = 0,0950 + \\ + 0,5994E - 0,0055E^2,$$

where E - energy of electron in MeV.

It is obvious that calculated distribution in silicon can be used for approximate estimate of quantity of defects, which arose after passage of protection from Al with thickness of x g/cm². The procedure of calculation of defects for the monoenergetic electrons easily is generalized to the case of the initial spectrum of electrons. Furthermore, instead of the experimental section can be used theoretical sections. From the comparison of the results of such calculations with the experiment it is possible to explain the validity of one or the other theoretical assumptions. Let us note that the more correct calculation of distribution of defects in the sample is obtained during the direct calculation of a quantity of defects in each segment of the traced trajectory.

§ 6. Consecutive account of secondary radiation.

Efficiency of diagrams of Monte Carlo examined for solving of

great circle of problems was demonstrated only in application to primary emission (electrons).

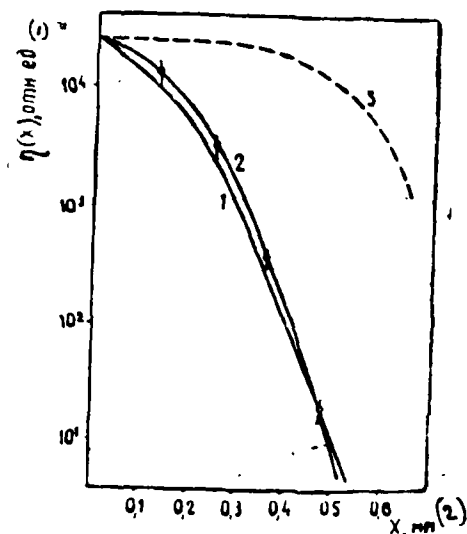


Fig. 36. Distribution in depth of defects $\eta(x)$ in Si, formed by electrons $E_0=0.5$ MeV. 1 - calculation by Monte Carlo; 2 - experiment; 3 - analytical calculation.

Key: (1). rel. un. (2). mm.

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Of no less interest for the practice are the problems, in which is required the detailed information about the characteristics of secondary radiation and, first of all, braking and X-radiation, which emerge from the target, bombarded with electrons ¹.

FOOTNOTE ¹. We do not examine the case of the very high energies, which initiate electron-photon cascade/stage. Similar problems were solved both by the analytical methods [124] and with the help of the method of Monte Carlo [125]. ENDFOOTNOTE.

It is known that the complete examination of these problems by contemporary theory is difficult.

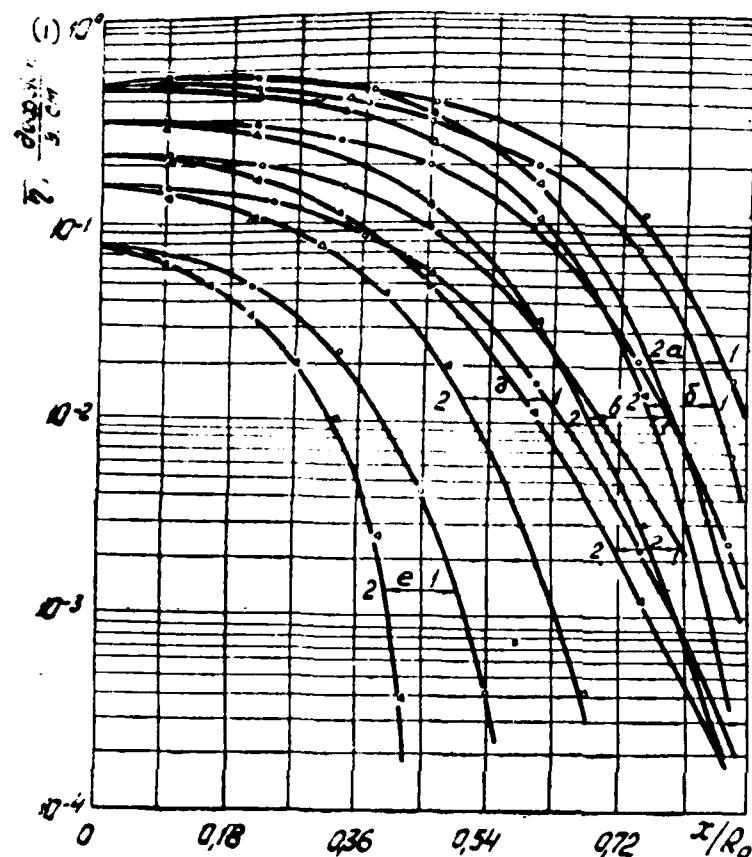


Fig. 37. Distribution of probability of formation of defects $G_{R_0}^x$ in Si for different wave energies of electron. 1 - defects, formed in Si. under layer C; 2 - defects in Si. a - $E_0=6$ MeV; b - $E_0=4$ MeV; c - $E_0=2$ MeV; d - $E_0=1$ MeV; e - $E_0=0.7$ MeV; f - $E_0=0.4$ MeV.

Key: (1). defect/e. cm.

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Here also the significant role play boundary conditions. In this case in the thick targets is substantial repeated scattering and fluctuation in the energy losses of primary electrons as well as multiple scattering and self-absorption of secondary radiation. At

present only with the help of the Monte Carlo method it is possible to obtain in entire space information about secondary radiation ¹.

FOOTNOTE ¹. Most efficient in this case proves to be the calculation procedure according to the method "catastrophic" collisions.

ENDFOOTNOTE.

The report of Ruffle [126] is one of the first works, carried out in this direction. In the process of tracking the generation of secondary electrons, positrons and γ -quanta was examined. For the drawing of photon energy differential cross section (1.31) (case of complete shielding $\gamma=0$) was used. During the conversion of γ -quanta into the pair electron - positron, the formation of Compton and photoelectrons the drawings were produced from the appropriate distributions. For the photoelectrons is used Sauter's formula [117]. Drawing of energy (angle) of the secondary electron of was produced from the distribution of Moller. The tracking of electrons was conducted in the approximation/approach of the continuous energy losses, angular distribution in multiple scattering was received as Gaussian. In the calculation the annihilation of positrons and the photoproduction of neutrons were taken into consideration also. Obtaining heat distribution, which is isolated in the target, irradiated by electrons with the energy in the interval of 50-150 MeV is the fundamental result of calculation.

Fundamental research of applicability of Monte Carlo method to

problem in question was carried out by Berger [127]. The distribution of intensity calculated by it in the spectrum of bremsstrahlung in incident direction in the electron is equal in Figure 38 with the experimental data of Rester etc. [128]. Agreement is very good and, which is especially valuable, even in the region, close to the maximum photon energy. The interesting calculation of braking electron beam by energy to 60 MeV in the thick target from lead led bibliophiles [129]. The drawings of energy of bremsstrahlung were conducted with the use of differential cross section of Shiff in the form (1.33), which ensures better agreement with the experiment in the high-energy region of γ -quanta, they are close to the boundary. Calculation was divided/marked off into two stages. In the first were computed the characteristics of the generated γ -quanta and electrons in each of 25 layers, to which was decomposed the target. Subsequently the tracking of secondary radiation, which leads to the appearance of tertiary generation of the particles, was conducted: electron-positron pairs; with photo- and Compton-electrons, etc.

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Electronic (positron) trajectories were outlined just as into [126], using an assumption about the continuous energy losses. The designed heat release will agree well with the experiment. It is possible to hope that these, until scarce, calculations are soon considerably expanded, and practice will obtain the reliable recommendations, necessary during the design of accelerators, conducting the calculations of protection, also, for purposes of technology. The

problem of the beam shaping of monoenergetic highenergy neutrons is also of great interest. However, similar calculations very labor-consuming and for their realization are required the computer(s), which possess the large memory and the speed.

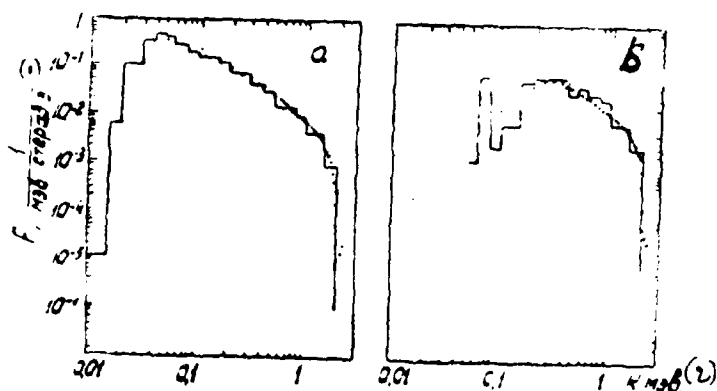


Fig. 38. Spectra of bremsstrahlung in Al (a), Au (b) $E_0=2$ MeV, $\psi=0^\circ$.

Histograms - calculation, point - experiment.

Key: (1). MeV sterad. 3. (2). MeV.

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Appendix.

Table 1-10.

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Table 1. Mott's factor R_M for electron scattering.

θ	$E_0, \text{ MeV } (1)$								
	10	4	2	1	0.7	0.4	0.2	0.1	0.05
$Z=6$									
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
15	0.999	0.999	1.000	1.000	1.001	1.002	1.003	1.004	1.004
30	0.961	0.962	0.963	0.967	0.970	0.977	0.987	0.996	1.001
45	0.888	0.890	0.894	0.903	0.911	0.929	0.954	0.976	0.990
60	0.788	0.790	0.797	0.814	0.828	0.860	0.906	0.946	0.974
75	0.666	0.669	0.680	0.705	0.728	0.776	0.846	0.909	0.953
90	0.532	0.537	0.551	0.586	0.617	0.683	0.781	0.868	0.930
105	0.397	0.403	0.421	0.466	0.505	0.590	0.714	0.826	0.905
120	0.269	0.277	0.298	0.352	0.440	0.501	0.651	0.786	0.882
135	0.1591	0.1680	0.1923	0.254	0.308	0.425	0.596	0.751	0.862
150	0.0742	0.0839	0.1106	0.1787	0.238	0.366	0.554	0.725	0.846
165	0.0206	0.0310	0.0591	0.1310	0.1938	0.328	0.527	0.708	0.836
180	0.0024	0.0129	0.0416	0.1148	0.1786	0.316	0.518	0.702	0.833
$Z=13$									
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
15	1.020	1.020	1.020	1.020	1.019	1.019	1.018	1.015	1.013
30	0.997	0.998	0.999	1.001	1.003	1.008	1.013	1.017	1.017
45	0.935	0.936	0.939	0.947	0.954	0.968	0.985	1.004	1.013
60	0.834	0.840	0.846	0.861	0.874	0.903	0.943	0.977	0.994
75	0.714	0.718	0.727	0.752	0.773	0.818	0.883	0.941	0.979
90	0.575	0.580	0.594	0.628	0.658	0.722	0.815	0.898	0.955
105	0.431	0.438	0.455	0.499	0.538	0.621	0.743	0.852	0.928

Key: (1). MeV.

Continuation of Table 1.

n	E. M. ϕ								
	10	4	2	1	0.7	0.4	0.2	0.1	0.05
120	0.294	0.302	0.323	0.377	0.424	0.525	0.674	0.808	0.902
135	0.1742	0.1831	0.207	0.270	0.324	0.441	0.613	0.769	0.879
150	0.0813	0.0912	0.1179	0.1868	0.247	0.376	0.566	0.738	0.861
165	0.0226	0.0329	0.0614	0.1342	0.1980	0.334	0.536	0.719	0.850
180	0.0024	0.0131	0.0419	0.1163	0.1811	0.320	0.526	0.712	0.846
Z = 29									
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
15	1.069	1.068	1.068	1.066	1.064	1.059	1.050	1.038	1.026
30	1.084	1.083	1.083	1.082	1.081	1.089	1.082	1.070	1.054
45	1.066	1.066	1.068	1.072	1.074	1.080	1.085	1.082	1.072
60	0.987	0.988	0.993	1.004	1.013	1.032	1.057	1.073	1.076
75	0.865	0.868	0.876	0.896	0.914	0.952	1.004	1.045	1.067
90	0.712	0.716	0.729	0.761	0.788	0.847	0.921	1.003	1.047
105	0.543	0.549	0.566	0.610	0.648	0.729	0.848	0.952	1.022
120	0.375	0.383	0.405	0.460	0.508	0.611	0.763	0.900	0.995
135	0.224	0.234	0.259	0.325	0.382	0.505	0.687	0.852	0.970
150	0.1052	0.1157	0.1446	0.218	0.282	0.420	0.626	0.813	0.950
165	0.0289	0.0402	0.0710	0.1497	0.218	0.366	0.587	0.789	0.937
180	0.0026	0.0141	0.0456	0.1261	0.1964	0.348	0.573	0.780	0.932
Z = 50									
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
15	1.124	1.123	1.121	1.115	1.109	1.096	1.072	1.043	1.018
30	1.235	1.234	1.231	1.123	1.216	1.197	1.161	1.113	1.060
45	1.292	1.292	1.290	1.284	1.279	1.264	1.232	1.183	1.120
60	1.274	1.274	1.275	1.277	1.277	1.276	1.262	1.228	1.173
75	1.177	1.179	1.184	1.197	1.208	1.227	1.246	1.242	1.207
90	1.013	1.017	1.028	1.055	1.078	1.126	1.188	1.226	1.225
105	0.801	0.807	0.825	0.869	0.907	0.988	1.101	1.190	1.230
120	0.569	0.578	0.602	0.664	0.718	0.832	0.999	1.142	1.229
135	0.348	0.359	0.390	0.467	0.535	0.682	0.899	1.093	1.225
150	0.1651	0.1781	0.214	0.305	0.385	0.537	0.815	1.053	1.221
165	0.0452	0.0694	0.0983	0.1980	0.285	0.475	0.760	1.026	1.219
180	0.0033	0.0180	0.0581	0.1608	0.251	0.446	0.741	1.017	1.219
Z = 82									
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
15	1.127	1.125	1.120	1.108	1.098	1.074	1.042	1.024	1.024
30	1.358	1.354	1.344	1.315	1.290	1.230	1.133	1.040	0.996
45	1.658	1.653	1.638	1.599	1.564	1.479	1.328	1.157	1.023
60	1.918	1.912	1.897	1.857	1.819	1.728	1.555	1.336	1.112
75	2.044	2.040	2.029	2.000	1.971	1.896	1.741	1.518	1.267
90	1.981	1.980	1.979	1.974	1.966	1.936	1.844	1.672	1.435
105	1.726	1.731	1.745	1.777	1.801	1.842	1.859	1.786	1.614
120	1.324	1.335	1.366	1.444	1.510	1.640	1.799	1.866	1.799
135	0.855	0.874	0.924	1.050	1.159	1.385	1.698	1.920	1.978
150	0.422	0.446	0.513	0.683	0.830	1.143	1.592	1.955	2.180
165	0.1158	0.1444	0.222	0.422	0.595	0.969	1.514	1.974	2.233
180	0.0068	0.0368	0.1187	0.328	0.511	0.908	1.486	1.978	2.267
Z = 92									
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
15	1.103	1.102	1.097	1.086	1.076	1.058	1.038	1.034	1.029
30	1.321	1.316	1.304	1.270	1.240	1.174	1.075	1.005	1.007
45	1.702	1.694	1.674	1.619	1.569	1.456	1.267	1.080	0.978
60	2.119	2.110	2.085	2.019	1.958	1.815	1.561	1.272	1.038
75	2.418	2.410	2.388	2.326	2.268	2.127	1.859	1.521	1.193
90	2.462	2.477	2.465	2.429	2.393	2.295	2.083	1.770	1.413
105	2.264	2.266	2.272	2.284	2.290	2.280	2.198	2.991	1.879
120	1.797	1.809	1.840	1.918	1.981	2.096	2.206	2.177	1.979
135	1.191	1.213	1.273	1.423	1.551	1.809	2.141	2.226	2.287
150	0.598	0.629	0.715	0.932	1.120	1.510	2.048	2.436	2.561
165	0.1652	0.203	0.307	0.572	0.841	1.287	1.970	2.505	2.753
180	0.0031	0.0496	0.1600	0.441	0.685	1.206	1.940	2.523	2.826

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Table 2. Values of parameter $\cos \gamma = f(q)$.

q	$\cos \gamma$	q	$\cos \gamma$
0	1,000	0,60	0,5290
0,05	0,9905	0,60	0,4471
0,10	0,9631	0,80	0,3323
0,15	0,9208	1,00	0,2610
0,20	0,8680	1,20	0,2145
0,30	0,7478	1,50	0,1696
0,40	0,6303	2,00	0,1261

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Table 3. Average/mean energy losses to the ionization over a unit of distance $-\frac{1}{\rho} \frac{dE}{dx}$ (MeV·cm²/g).

Z	E_0 Mev (1)										
	0,1	0,2	0,4	0,6	0,8	1,0	2,0	3,0	4,0	5,0	6,0
3,33	4,3116	2,9098	2,2264	2,0326	1,9583	1,9286	1,9448	2,0029	2,0564	2,1026	2,1424
3,50	4,2495	2,8692	2,1962	2,0055	1,9325	1,9034	1,9202	1,9779	2,0310	2,0767	2,1163
6,0	3,6871	2,5032	1,9256	1,7633	1,7024	1,6792	1,7013	1,7565	1,8065	1,8492	1,8861
13,0	3,1780	2,1792	1,6913	1,5565	1,5079	1,4912	1,5223	1,5780	1,6271	1,6689	1,7047
14,0	3,2380	2,2235	1,7279	1,5913	1,5423	1,5259	1,5593	1,6172	1,6682	1,7115	1,7486
22,0	2,7988	1,9339	1,5111	1,3958	1,3556	1,3432	1,3788	1,4334	1,4808	1,5209	1,5553
26,0	2,8170	1,9479	1,5230	1,4072	1,3671	1,3548	1,3911	1,4468	1,4950	1,5356	1,5704
29,0	2,6273	1,8260	1,4341	1,3283	1,2925	1,2825	1,3218	1,3771	1,4246	1,4647	1,4989
32,0	2,5073	1,7449	1,3719	1,2715	1,2377	1,2285	1,2673	1,3208	1,3668	1,4056	1,4386
42,0	2,3557	1,6494	1,3035	1,2115	1,1816	1,1744	1,2164	1,2704	1,3165	1,3551	1,3880
47,0	2,3115	1,6210	1,2828	1,1931	1,1643	1,1576	1,2033	1,2543	1,3002	1,3387	1,3715
50,0	2,1677	1,5255	1,2108	1,1280	1,1018	1,0965	1,1394	1,1921	1,2366	1,2739	1,3057
82,0	1,9316	1,3680	1,0916	1,0197	0,9980	0,9945	1,0376	1,0877	1,1298	1,1650	1,1950

Key: (1). MeV.

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Table 4. Average/mean total losses of energy over a unit of distance

$$\left(-\frac{1}{E} \frac{dE}{dx} \right) \text{ (Mev} \cdot \text{cm}^2/\text{g)}.$$

Z	$E_0, \text{ Mev (1)}$										
	0.1	0.2	0.4	0.6	0.8	1.0	2.0	3.0	4.0	5.0	6.0
3.33	4.3116	2.9098	2.2264	2.0358	1.9626	1.9340	1.9570	2.0220	2.0849	2.1401	2.1894
3.50	4.2495	2.8692	2.1962	2.0088	1.9369	1.9091	1.9328	1.9987	2.0606	2.1158	2.1653
6.0	3.6871	2.5032	1.9256	1.7685	1.7094	1.6882	1.7215	1.7896	1.8536	1.9110	1.9640
13.0	3.1780	2.1792	1.6913	1.5673	1.5225	1.5099	1.5644	1.6471	1.7257	1.7988	1.8676
14.0	3.2380	2.2235	1.7279	1.6033	1.5586	1.5467	1.6062	1.6942	1.7780	1.8562	1.9300
22.0	2.7988	1.9839	1.5111	1.4132	1.3792	1.3734	1.4467	1.5448	1.6398	1.7305	1.8180
26.0	2.8170	1.9479	1.5230	1.4281	1.3953	1.3909	1.4728	1.5803	1.6853	1.7866	1.8851
29.0	2.6273	1.8260	1.4341	1.3511	1.3234	1.3220	1.4108	1.5230	1.6327	1.7391	1.8429
32.0	2.5073	1.7449	1.3719	1.2958	1.2706	1.2706	1.3621	1.4764	1.5887	1.6981	1.8054
42.0	2.3557	1.6494	1.3035	1.2431	1.2244	1.2293	1.3400	1.4732	1.6056	1.7364	1.8660
47.0	2.3115	1.6210	1.2828	1.2284	1.2120	1.2188	1.3380	1.4801	1.6223	1.7634	1.9039
50.0	2.1677	1.5255	1.2108	1.1642	1.1510	1.1593	1.2810	1.4243	1.5679	1.7107	1.8533
82.0	1.9816	1.3880	1.0916	1.0756	1.0737	1.0914	1.2558	1.4456	1.6402	1.8380	2.0386

Key: (1). MeV.

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Table 5. Values of complete landing runs R_0 , g/cm².

Z	E_0 , Mev (1)										
	0.1	0.2	0.4	0.5	0.8	1.0	2.0	3.0	4.0	5.0	6.0
3.33	0.0136	0.0429	0.1238	0.2185	0.3188	0.4216	0.8284	1.4410	1.9278	2.4010	2.8629
3.50	0.0138	0.0435	0.1256	0.2215	0.3232	0.4273	0.9507	1.4595	1.9520	2.4208	2.8979
6.0	0.0160	0.0502	0.1440	0.2532	0.3685	0.4864	1.0759	1.6456	2.1944	2.7255	3.2415
13.0	0.0188	0.0582	0.1654	0.2891	0.4189	0.5510	1.2046	1.8274	2.4203	2.9877	3.5333
14.0	0.0185	0.0571	0.1621	0.2831	0.4100	0.5389	1.1762	1.7823	2.3582	2.9084	3.4360
22.0	0.0215	0.0660	0.1864	0.3243	0.4679	0.6133	1.3253	1.9944	2.6224	3.2158	3.7794
26.0	0.0214	0.0656	0.1850	0.3217	0.4638	0.6075	1.3091	1.9643	2.5767	3.1525	3.6974
29.0	0.0231	0.0704	0.1974	0.3422	0.4921	0.6435	1.3786	2.0605	2.6943	3.2874	3.8458
32.0	0.0242	0.0737	0.2066	0.3578	0.5140	0.6716	1.4347	2.1395	2.7921	3.4006	3.9715
42.0	0.0260	0.0785	0.2186	0.3771	0.5395	0.7027	1.4846	2.1960	2.8457	3.4443	4.0000
47.0	0.0265	0.0800	0.2225	0.3832	0.5474	0.7121	1.4980	2.2081	2.8530	3.4439	3.9894
50.0	0.0284	0.0853	0.2364	0.4064	0.5795	0.7528	1.5761	2.3160	2.9846	3.5948	4.1561
82.0	0.0322	0.0958	0.2638	0.4504	0.6368	0.8216	1.6784	2.4200	3.0688	3.6442	4.1804

Key: (1). MeV.

Page 6. Integral function of the energy-loss distribution on Simon.

$\frac{\Delta E - \Delta E_b}{\Delta_0}$	$\Sigma_c(E)$	$\Sigma_c(E)$	$\Sigma_c(E)$	$\Sigma_c(E)$	$\Sigma_c(E)$	$\Sigma_c(E)$	$\Sigma_c(E)$
100	0.00846	8.500	0.09847	4.750	0.17100	1.000	0.49609
95	0.00890	8.375	0.09983	4.625	0.17535	0.875	0.52017
90	0.00940	8.250	0.10123	4.500	0.17993	0.750	0.54558
85	0.00994	8.125	0.10270	4.375	0.18474	0.625	0.57243
80	0.01058	8.000	0.10421	4.250	0.18979	0.500	0.60087
75	0.01128	7.875	0.10577	4.125	0.19510	0.375	0.63087
70	0.01209	7.750	0.10736	4.000	0.20071	0.250	0.66193
65	0.01302	7.625	0.10902	3.875	0.20659	0.125	0.69365
60	0.01410	7.500	0.11072	3.750	0.21280	0.000	0.72574
55	0.01539	7.375	0.11250	3.625	0.21932	-0.125	0.75789
50	0.01693	7.250	0.11336	3.500	0.22621	-0.250	0.78984
45	0.01880	7.125	0.11624	3.375	0.23350	-0.375	0.82133
40	0.02116	7.000	0.11820	3.250	0.24119	-0.500	0.85203
35	0.02417	6.875	0.12024	3.125	0.24933	-0.625	0.88086
30	0.02821	6.750	0.12233	3.000	0.25795	-0.750	0.90683
25	0.03386	6.625	0.12450	2.875	0.26703	-0.875	0.92948
20	0.04232	6.500	0.12677	2.750	0.27674	-1.000	0.94797
15	0.05642	6.375	0.12913	2.625	0.28701	-1.125	0.96250
10	0.08463	6.250	0.13158	2.500	0.29790	-1.250	0.97376
9.875	0.08568	6.125	0.13415	2.375	0.30946	-1.375	0.98233
9.750	0.08672	6.000	0.13683	2.250	0.32175	-1.500	0.98858
9.625	0.08780	5.875	0.13962	2.125	0.33482	-1.625	0.9927
9.500	0.08888	5.750	0.14248	2.000	0.34870	-1.750	0.99547
9.375	0.08999	5.625	0.14555	1.875	0.36343	-1.875	0.99734
9.250	0.09111	5.500	0.14877	1.750	0.37903	-2.000	0.99861
9.125	0.09227	5.375	0.15201	1.625	0.39564	-2.125	0.99943
9.000	0.09346	5.250	0.15547	1.500	0.41328	-2.250	0.99988
8.875	0.09466	5.125	0.15911	1.375	0.43206	-2.375	1.00000
8.750	0.09589	5.000	0.16290	1.250	0.45207	-2.500	1.00000
8.625	0.09716	4.875	0.16685	1.125	0.47337		

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USING THE MONTE CARLO METHOD TO SOLVE PROBLEMS OF THE

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TRANSFER OF FAST EL (U) FOREIGN TECHNOLOGY DIV

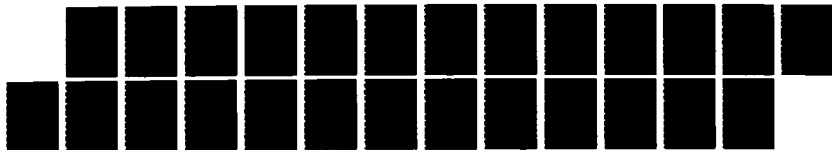
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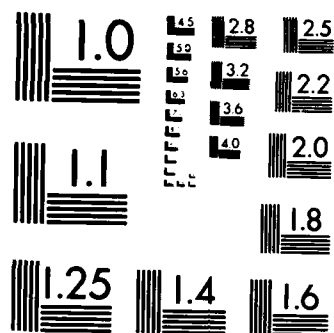
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Table 7. Values of Moliere-Bethe functions.

β	$f^{(0)}$	$f^{(1)}$	$f^{(2)}$
0	2	0,8458	2,4929
0,2	1,9216	0,7038	2,0694
0,4	1,7214	0,3437	1,0488
0,6	1,4094	-0,0777	-0,0044
0,8	1,0546	-0,3981	-0,6068
1,0	0,7338	-0,5285	-0,6359
1,2	0,4738	-0,4770	-0,3086
1,4	0,2817	-0,3183	0,0525
1,6	0,1546	-0,1396	0,2423
1,8	0,0783	-0,0006	0,2386
2,0	0,0366	0,0782	0,1316
2,2	0,01581	0,1054	0,0196
2,4	0,00630	0,1008	0,0467
2,6	0,00232	0,08282	0,0649
2,8	0,00079	0,06247	-0,0546
3,0	0,000250	0,04550	-0,03568
3,2	$7,3 \cdot 10^{-5}$	0,03288	-0,01923
3,4	$1,9 \cdot 10^{-5}$	0,02402	-0,00847
3,6	$4,7 \cdot 10^{-6}$	0,01791	-0,00264
3,8	$1,1 \cdot 10^{-6}$	0,01386	0,00005
4,0	0	$10,638 \cdot 10^{-3}$	$1,0741 \cdot 10^{-3}$
4,5	0	$6,140 \cdot 10^{-3}$	$1,2294 \cdot 10^{-3}$
5,0	0	$3,831 \cdot 10^{-3}$	$0,8326 \cdot 10^{-3}$
5,5	0	$2,527 \cdot 10^{-3}$	$0,5368 \cdot 10^{-3}$
6,0	0	$1,739 \cdot 10^{-3}$	$0,3495 \cdot 10^{-3}$
7,0	0	$0,908 \cdot 10^{-3}$	$0,1584 \cdot 10^{-3}$
8,0	0	$0,521 \cdot 10^{-3}$	$0,0783 \cdot 10^{-3}$
9,0	0	$0,3208 \cdot 10^{-3}$	$0,0417 \cdot 10^{-3}$
10,0	0	$0,2084 \cdot 10^{-3}$	$0,0237 \cdot 10^{-3}$

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Table 8. The angular distribution of multiple scattering in the theory of Nigem, etc. [58].

θ	B							
	4	5	6	7	8	9	10	12
0.0	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
0.2	0.94070	0.94546	0.94850	0.95058	0.95203	0.95321	0.95409	0.95537
0.4	0.78389	0.79992	0.81017	0.81721	0.82232	0.82616	0.82916	0.83351
0.6	0.58102	0.60800	0.62535	0.63731	0.64601	0.65259	0.65772	0.66520
0.8	0.38726	0.41889	0.43939	0.45363	0.46302	0.47192	0.47811	0.48716
1.0	0.23800	0.26632	0.28491	0.29793	0.30752	0.31486	0.32063	0.32913
1.2	0.14139	0.16116	0.17437	0.18377	0.19077	0.19616	0.20045	0.20681
1.4	0.086499	0.096809	0.10393	0.10911	0.11304	0.11612	0.11859	0.12231
1.6	0.056661	0.059859	0.062258	0.064103	0.065557	0.066729	0.067692	0.069178
1.8	0.038991	0.038401	0.038158	0.038067	0.038046	0.038059	0.038088	0.038165
2.0	0.026848	0.025061	0.023871	0.023029	0.022403	0.021920	0.021538	0.020972
2.2	0.017934	0.016280	0.015071	0.014158	0.013446	0.012878	0.012413	0.011702
2.4	0.011635	0.010476	0.009557	0.008826	0.008237	0.007754	0.007353	0.006726
2.6	0.007994	0.007163	0.006465	0.005894	0.005425	0.005038	0.004709	0.004193
2.8	0.005491	0.004893	0.004382	0.003960	0.003611	0.003323	0.003077	0.002690
3.0	0.003969	0.003492	0.003096	0.002772	0.002508	0.002291	0.002106	0.001816
3.2	0.002997	0.002592	0.002271	0.002016	0.001810	0.001644	0.001502	0.001283
3.4	0.002322	0.001977	0.001714	0.001510	0.001348	0.001219	0.001103	0.000940
3.6	0.001824	0.001535	0.001320	0.001156	0.001027	0.000926	0.000839	0.000708
3.8	0.001445	0.001206	0.001031	0.000900	0.000797	0.000718	0.000648	—
4.0	0.001153	0.000957	0.000816	0.000710	0.000628	0.000565	0.000510	—
4.2	0.000928	0.000768	0.000654	0.000568	0.000502	0.000452	0.000407	—
4.4	0.000754	0.000623	0.000530	0.000460	0.000406	0.000366	0.000329	—
4.6	0.000618	0.000511	0.000434	0.000377	0.000333	0.000300	0.000269	—
4.8	0.000512	0.000423	0.000359	0.000312	0.000275	0.000249	0.000223	—
5.0	0.000427	0.000353	0.000300	0.000260	0.000230	0.000208	0.000186	—
5.2	0.000360	0.000297	0.000252	0.000219	0.000194	0.000176	0.000156	—
5.4	0.000305	0.000252	0.000214	0.000186	0.000164	0.000150	0.000133	—
5.6	0.000261	0.000215	0.000183	0.000159	0.000140	0.000129	—	—
5.8	0.000224	0.000185	0.000157	0.000137	0.000121	0.000111	—	—
6.0	0.000194	0.000160	0.000136	0.000118	0.000104	0.000097	—	—
6.2	0.000168	0.000139	0.000118	0.000103	0.000091	0.000085	—	—
6.4	0.000147	0.000122	0.000103	0.000090	0.000079	0.000075	—	—
6.6	0.000129	0.000107	0.000091	0.000079	0.000070	0.000066	—	—
6.8	0.000114	0.000094	0.000080	0.000070	0.000062	0.000059	—	—
7.0	0.000101	0.000083	0.000071	0.000062	0.000055	0.000053	—	—
7.2	0.000090	0.000074	0.000063	0.000055	0.000049	0.000048	—	—
7.4	0.000080	0.000066	0.000056	0.000049	0.000043	0.000043	—	—
7.6	0.000071	0.000059	0.000050	0.000044	0.000039	0.000040	—	—
7.8	0.000064	0.000053	0.000045	0.000039	0.000035	0.000036	—	—
8.0	0.000058	0.000048	0.000041	0.000035	0.00003	0.000034	—	—

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Table 9. Coefficients of transmission T_N and T_E for Plexiglas of $\psi_0=0^\circ$ [d (g/cm²) the value of space in the depth].

$E_0=6.0 \ d=0.146$		$E_0=4.0 \ d=0.098$		$E_0=2.0 \ d=0.0475$		$E_0=1.0 \ d=0.0213$		$E_0=0.7 \ d=0.0136$		$E_0=0.4 \ d=0.00625$	
T_N	T_E	T_N	T_E	T_N	T_E	T_N	T_E	T_N	T_E	T_N	T_E
$\psi_0=0$											
0.996	0.934	0.997	0.938	0.997	0.946	0.997	0.950	0.999	0.958	0.998	0.958
0.992	0.875	0.992	0.883	0.994	0.890	0.993	0.903	0.995	0.914	0.993	0.915
0.989	0.817	0.988	0.830	0.988	0.834	0.987	0.853	0.985	0.863	0.984	0.864
0.985	0.760	0.984	0.778	0.980	0.778	0.979	0.801	0.981	0.815	0.978	0.821
0.975	0.693	0.974	0.717	0.967	0.720	0.971	0.747	0.969	0.759	0.963	0.763
0.965	0.642	0.967	0.661	0.947	0.659	0.962	0.690	0.945	0.693	0.946	0.705
0.958	0.588	0.956	0.605	0.928	0.598	0.940	0.628	0.918	0.631	0.912	0.641
0.943	0.529	0.944	0.548	0.906	0.534	0.911	0.559	0.875	0.557	0.870	0.573
0.927	0.474	0.923	0.487	0.872	0.467	0.863	0.486	0.829	0.485	0.818	0.499
0.908	0.418	0.893	0.426	0.824	0.398	0.796	0.410	0.770	0.410	0.765	0.426
0.884	0.360	0.858	0.364	0.762	0.333	0.719	0.335	0.706	0.336	0.680	0.344
0.834	0.301	0.815	0.300	0.697	0.268	0.636	0.265	0.612	0.266	0.597	0.278
0.774	0.239	0.737	0.239	0.600	0.203	0.544	0.199	0.507	0.199	0.517	0.217
0.696	0.184	0.646	0.178	0.501	0.148	0.448	0.143	0.405	0.142	0.426	0.160
0.601	0.131	0.526	0.124	0.381	0.099	0.322	0.091	0.312	0.095	0.317	0.107
0.469	0.094	0.406	0.080	0.280	0.060	0.214		0.212		0.219	0.065
0.311		0.262		0.161		0.129		0.131		0.129	
0.179		0.153		0.087		0.071					

Continuation of Table 9.

$E_0 = 6.0 \quad d = 0.146$		$E_0 = 4.0 \quad d = 0.098$		$E_0 = 2.0 \quad d = 0.0475$		$E_0 = 1.0 \quad d = 0.0213$		$E_0 = 0.7 \quad d = 0.0136$		$E_0 = 0.4 \quad d = 0.0062$	
T_N	T_E	T_N	T_E	T_N	T_E	T_N	T_E	T_N	T_E	T_N	T_E
0.995	0.929	0.994	0.928	0.998	0.941	0.996	0.943	0.999	0.948	0.998	0.952
0.992	0.865	0.991	0.863	0.988	0.876	0.986	0.885	0.988	0.893	0.985	0.902
0.991	0.802	0.988	0.800	0.979	0.808	0.977	0.825	0.979	0.839	0.979	0.851
0.983	0.737	0.980	0.738	0.971	0.745	0.957	0.756	0.964	0.774	0.966	0.790
0.975	0.671	0.973	0.679	0.944	0.673	0.936	0.687	0.944	0.703	0.935	0.720
0.965	0.608	0.959	0.610	0.921	0.600	0.903	0.615	0.909	0.627	0.899	0.643
0.955	0.543	0.943	0.545	0.886	0.527	0.862	0.540	0.851	0.545	0.850	0.565
0.934	0.475	0.919	0.479	0.845	0.454	0.819	0.468	0.796	0.470	0.801	0.492
0.903	0.408	0.882	0.407	0.785	0.381	0.751	0.391	0.742	0.400	0.741	0.417
0.857	0.341	0.829	0.340	0.713	0.309	0.674	0.318	0.672	0.332	0.658	0.342
0.797	0.276	0.761	0.272	0.630	0.243	0.574	0.249	0.587	0.260	0.568	0.275
0.736	0.217	0.665	0.207	0.548	0.186	0.499	0.190	0.494	0.196	0.484	0.212
0.634	0.159	0.571	0.152	0.441	0.133	0.414	0.140	0.397	0.139	0.390	0.156
0.516	0.109	0.450	0.103	0.349	0.091	0.303	0.092	0.291	0.090	0.302	0.109
0.380		0.334		0.249		0.227		0.197		0.217	
0.238		0.219		0.166		0.135		0.125		0.148	
0.118		0.130		0.078		0.071		0.071		0.036	
$\psi = 45^\circ$											
0.993	0.909	0.998	0.917	0.996	0.923	0.995	0.929	0.993	0.930	0.996	0.936
0.988	0.826	0.990	0.838	0.990	0.847	0.982	0.850	0.978	0.854	0.973	0.857
0.978	0.742	0.978	0.759	0.971	0.755	0.949	0.755	0.931	0.755	0.934	0.766
0.958	0.654	0.965	0.677	0.924	0.647	0.903	0.656	0.884	0.660	0.876	0.666
0.938	0.569	0.941	0.584	0.878	0.553	0.851	0.567	0.835	0.572	0.823	0.574
0.914	0.485	0.899	0.492	0.825	0.467	0.787	0.477	0.766	0.480	0.732	0.473
0.865	0.397	0.846	0.407	0.753	0.383	0.710	0.391	0.685	0.394	0.662	0.392
0.789	0.314	0.762	0.323	0.672	0.311	0.634	0.317	0.607	0.316	0.589	0.322
0.704	0.237	0.685	0.250	0.579	0.242	0.557	0.252	0.528	0.250	0.512	0.255
0.596	0.175	0.589	0.189	0.504	0.188	0.468	0.190	0.458	0.194	0.433	0.197
0.494	0.125	0.485	0.136	0.418	0.137	0.380	0.139	0.372	0.144	0.350	0.146
0.380	0.082	0.383	0.092	0.336	0.096	0.299	0.096	0.302	0.104	0.261	0.100
0.269		0.297		0.256		0.216		0.221		0.194	
0.174		0.199		0.151		0.151		0.142		0.136	
0.099		0.114		0.098		0.099		0.089		0.082	
$\psi = 63^\circ$											
0.988	0.873	0.991	0.866	0.968	0.856	0.964	0.862	0.954	0.855	0.950	0.862
0.978	0.739	0.975	0.737	0.894	0.684	0.862	0.672	0.853	0.671	0.862	0.700
0.928	0.586	0.909	0.578	0.798	0.536	0.769	0.527	0.760	0.533	0.757	0.558
0.839	0.440	0.807	0.433	0.729	0.426	0.681	0.421	0.681	0.426	0.682	0.459
0.755	0.332	0.719	0.326	0.656	0.335	0.600	0.334	0.591	0.335	0.586	0.363
0.655	0.239	0.601	0.231	0.578	0.259	0.536	0.267	0.502	0.261	0.516	0.295
0.550	0.169	0.493	0.165	0.497	0.198	0.452	0.205	0.410	0.196	0.448	0.234
0.432	0.112	0.409	0.116	0.394	0.145	0.376	0.153	0.344	0.151	0.385	0.184
0.321	0.072	0.321	0.077	0.326	0.106	0.298	0.111	0.276	0.108	0.323	0.140
0.208		0.228		0.242	0.073	0.226	0.077	0.214	0.077	0.254	0.102
0.139		0.143		0.179		0.166		0.159		0.200	
0.096		0.083		0.129		0.113		0.116		0.142	
$\psi = 81^\circ$											
0.805	0.522	0.773	0.504	0.673	0.498	0.609	0.472	0.625	0.490	0.623	0.497
0.619	0.293	0.588	0.282	0.547	0.329	0.501	0.328	0.522	0.348	0.506	0.350
0.468	0.181	0.477	0.187	0.463	0.236	0.435	0.250	0.433	0.256	0.438	0.270
0.361	0.117	0.370	0.125	0.379	0.168	0.362	0.184	0.356	0.190	0.370	0.205
0.281	0.075	0.279	0.083	0.303	0.123	0.300	0.138	0.292	0.242	0.303	0.154
0.195		0.202		0.243	0.087	0.254	0.104	0.241	0.105	0.252	0.114
0.123		0.145		0.186		0.198	0.071	0.193	0.078	0.191	0.082
0.079		0.099		0.135		0.142		0.152		0.144	
				0.097		0.102		0.110		0.107	

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Energy spectra of electrons passing through flat layers x , g/cm^2 . $r/cm^2 = g/cm^2$; $M_{\text{эв}} = \text{MeV}$.

$n=9$	$n=8$	$n=7$	$n=6$	$n=5$	$n=4$	$n=3$	$n=2$	$n=1$	$n=0$	$x, g/cm^2$	$E_{\text{max}}, \text{MeV}$
$E_0 = 6.0 \text{ MeV} \quad \psi_0 = 0^\circ$											
1.633	0.109	0.056	0.019	0.007	0.002	0.002	0.002	0.002	—	0.396	5.468
1.466	0.340	0.121	0.053	0.027	0.018	0.009	0.002	—	—	0.792	4.867
1.139	0.727	0.205	0.145	0.080	0.045	0.025	0.010	0.005	—	1.189	4.201
0.687	0.880	0.500	0.245	0.132	0.088	0.052	0.038	0.025	0.012	1.486	3.769
0.046	0.832	0.740	0.496	0.324	0.199	0.139	0.092	0.060	0.023	1.783	3.409
0.023	0.128	0.785	0.728	0.553	0.396	0.274	0.175	0.093	0.029	2.081	3.147
0.008	0.066	0.597	0.785	0.702	0.611	0.504	0.306	0.190	0.058	2.378	2.625
0.016	0.049	0.214	0.741	0.923	0.956	0.873	0.610	0.313	0.066	2.675	2.104
0.037	0.073	0.257	0.588	0.955	1.028	0.955	0.771	0.551	0.220	2.873	1.843
$E_0 = 6.0 \text{ MeV} \quad \psi_0 = 27^\circ$											
1.640	0.118	0.016	0.013	0.007	0.002	0.002	0.002	0.002	—	0.297	5.550
1.492	0.292	0.094	0.052	0.016	0.009	0.008	0.006	0.006	0.004	0.594	5.087
0.911	0.772	0.221	0.116	0.065	0.033	0.016	0.012	0.007	0.002	0.891	4.640
0.333	1.001	0.516	0.242	0.120	0.066	0.041	0.029	0.020	0.010	1.189	4.203
0.173	0.769	0.687	0.412	0.256	0.154	0.102	0.060	0.033	0.014	1.486	3.762
0.255	0.655	0.766	0.522	0.372	0.261	0.172	0.105	0.061	0.017	1.783	3.141
0.268	0.498	0.632	0.640	0.536	0.439	0.357	0.223	0.171	0.052	2.081	2.621
0.216	0.468	0.612	0.684	0.732	0.720	0.612	0.432	0.228	0.060	2.378	2.106
0.203	0.565	0.802	0.921	1.010	1.040	0.821	0.565	0.267	0.060	2.675	1.585
$E_0 = 6.0 \text{ MeV} \quad \psi_0 = 45^\circ$											
1.634	0.122	0.036	0.013	0.011	0.009	0.006	0.006	0.004	0.002	0.297	5.430
1.134	0.627	0.148	0.069	0.034	0.013	0.013	0.011	0.006	0.004	0.594	4.850
0.287	0.910	0.554	0.198	0.135	0.064	0.046	0.040	0.020	0.008	0.891	4.427
0.091	0.534	0.623	0.480	0.328	0.200	0.120	0.077	0.040	0.014	1.189	3.986
0.037	0.301	0.514	0.558	0.529	0.367	0.250	0.154	0.083	0.026	1.486	3.549
0.009	0.089	0.364	0.550	0.603	0.568	0.461	0.310	0.178	0.053	1.783	3.141
0.015	0.061	0.273	0.546	0.591	0.621	0.651	0.545	0.333	0.182	2.081	2.621
0.037	0.257	0.624	0.918	0.918	0.845	0.734	0.588	0.367	0.147	2.378	1.843
$E_0 = 6.0 \text{ MeV} \quad \psi_0 = 63^\circ$											
1.482	0.296	0.084	0.033	0.029	0.020	0.010	0.003	0.006	—	0.297	5.084
0.400	2.679	0.468	0.279	0.151	0.090	0.066	0.053	0.033	0.018	0.594	4.475
0.162	0.359	0.465	0.436	0.355	0.232	0.203	0.141	0.077	0.028	0.891	3.986
0.068	0.225	0.403	0.497	0.466	0.387	0.324	0.251	0.162	0.052	1.189	3.546
0.109	0.297	0.454	0.547	0.532	0.485	0.422	0.313	0.203	0.169	1.486	2.887
0.037	0.240	0.424	0.608	0.664	0.627	0.516	0.387	0.240	0.074	1.684	2.629
0.036	0.142	0.356	0.783	0.855	0.748	0.570	0.392	0.249	0.107	1.882	2.360
$E_0 = 6.0 \text{ MeV} \quad \psi_0 = 81^\circ$											
0.608	0.641	0.288	0.163	0.135	0.095	0.058	0.037	0.022	0.015	0.198	4.858
0.120	0.361	0.546	0.391	0.253	0.198	0.163	0.142	0.089	0.030	0.396	4.408
0.121	0.260	0.392	0.489	0.417	0.314	0.217	0.151	0.091	0.036	0.594	4.023
0.063	0.173	0.301	0.430	0.460	0.392	0.331	0.256	0.188	0.075	0.792	3.761
0.063	0.170	0.318	0.445	0.509	0.498	0.424	0.318	0.212	0.064	0.991	3.326
0.086	0.258	0.373	0.488	0.545	0.545	0.488	0.373	0.230	0.086	1.189	2.885
0.093	0.279	0.419	0.512	0.559	0.605	0.605	0.512	0.373	0.140	1.387	2.445
$E_0 = 4.0 \text{ MeV} \quad \psi_0 = 0^\circ$											
2.481	0.166	0.039	0.017	0.014	0.009	—	—	—	—	0.266	3.676
1.960	0.700	0.160	0.071	0.062	0.031	0.015	0.012	0.003	0.003	0.533	3.325
0.639	1.698	0.582	0.159	0.116	0.060	0.032	0.021	0.011	0.004	0.800	3.024
0.305	1.473	1.078	0.431	0.198	0.090	0.054	0.025	0.014	0.007	1.000	2.721
0.031	1.064	1.290	0.789	0.470	0.282	0.138	0.075	0.031	0.013	1.200	2.420
0.154	0.910	1.087	0.910	0.694	0.501	0.339	0.200	0.123	0.031	1.400	2.022
0.086	0.320	0.890	1.127	1.032	0.783	0.593	0.403	0.225	0.083	1.600	1.827
0.023	0.249	0.725	1.132	1.427	1.404	0.974	0.634	0.362	0.113	1.800	1.429
0.327	0.794	1.074	1.214	1.354	1.354	1.260	1.120	0.887	0.420	1.934	1.024

$E = 4.0 \text{ MeV}$ $\psi = 27^\circ$

2.472	0.135	0.043	0.022	0.005	0.003	0.003	0.003	0.003	—	0.200	3.728
2.228	0.456	0.110	0.053	0.044	0.018	0.012	0.003	—	—	0.400	3.427
1.233	1.273	0.319	0.163	0.093	0.047	0.017	0.007	0.003	—	0.600	3.173
0.688	1.465	0.692	0.230	0.176	0.111	0.061	0.034	0.023	0.004	0.800	2.874
0.301	1.092	1.024	0.594	0.329	0.213	0.153	0.104	0.060	0.020	1.000	2.572
0.263	0.818	0.953	0.826	0.586	0.428	0.300	0.195	0.105	0.030	1.200	2.220
0.032	0.428	0.834	1.016	0.887	0.684	0.503	0.342	0.182	0.043	1.400	2.021
0.100	0.478	1.016	1.155	1.055	0.856	0.677	0.478	0.279	0.080	1.600	1.629
0.047	0.199	0.425	0.993	1.276	1.276	1.182	0.945	0.567	0.142	1.800	1.429

 $E_0 = 4.0 \text{ MeV}$ $\psi = 45^\circ$

2.437	0.178	0.070	0.027	0.017	0.008	0.008	0.006	0.003	—	0.200	3.633
1.943	0.709	0.188	0.107	0.065	0.023	0.020	0.010	0.006	0.006	0.400	3.256
0.493	1.341	0.705	0.347	0.177	0.096	0.062	0.043	0.031	0.015	0.600	3.027
0.185	1.019	0.935	0.602	0.384	0.245	0.144	0.088	0.046	0.028	0.800	2.720
0.377	0.800	0.835	0.684	0.562	0.446	0.330	0.220	0.116	0.035	1.000	2.271
0.073	0.425	0.777	0.922	0.813	0.655	0.522	0.388	0.267	0.109	1.200	2.021
0.178	0.511	0.822	1.110	1.243	0.933	0.644	0.422	0.244	0.067	1.400	1.629
0.275	0.660	0.935	1.210	1.375	1.320	1.109	0.770	0.440	0.110	1.600	1.220

 $E_0 = 4.0 \text{ MeV}$ $\psi = 63^\circ$

2.181	0.423	0.140	0.039	0.027	0.024	0.024	0.024	0.012	0.006	0.200	3.453
0.523	1.144	0.683	0.296	0.189	0.136	0.123	0.082	0.053	0.017	0.400	3.087
0.049	0.493	0.822	0.707	0.493	0.362	0.257	0.175	0.100	0.027	0.600	2.877
0.098	0.377	0.694	0.747	0.656	0.553	0.445	0.317	0.181	0.060	0.800	2.428
0.063	0.277	0.517	0.769	0.946	0.946	0.563	0.353	0.202	0.076	1.000	2.124
0.115	0.345	0.575	0.863	1.007	0.949	0.777	0.518	0.259	0.086	1.133	1.824
0.103	0.257	0.463	0.772	1.080	1.235	1.235	0.926	0.463	0.154	1.267	1.620

 $E_0 = 4.0 \text{ MeV}$ $\psi = 81^\circ$

0.619	1.051	0.519	0.266	0.240	0.148	0.065	0.048	0.039	0.017	0.133	3.325
0.123	0.579	0.689	0.499	0.425	0.332	0.258	0.172	0.135	0.055	0.266	3.066
0.105	0.428	0.585	0.612	0.524	0.402	0.376	0.210	0.186	0.114	0.400	2.843
0.044	0.133	0.300	0.522	0.733	0.711	0.533	0.378	0.233	0.089	0.533	2.624
0.109	0.310	0.450	0.558	0.651	0.698	0.620	0.497	0.357	0.155	0.667	2.274
0.130	0.338	0.495	0.625	0.729	0.807	0.833	0.703	0.338	0.078	0.800	1.975
0.189	0.471	0.628	0.722	0.816	0.911	0.848	0.534	0.282	0.094	0.867	1.829

 $E_0 = 2.0 \text{ MeV}$ $\psi = 0^\circ$

4.833	0.242	0.047	0.037	0.021	0.021	0.016	0.005	0.005	—	0.097	1.913
4.550	0.705	0.149	0.057	0.052	0.034	0.023	0.011	0.006	0.006	0.195	1.788
2.891	1.996	0.551	0.241	0.103	0.069	0.055	0.041	0.028	0.014	0.292	1.670
1.862	2.517	1.104	0.493	0.228	0.155	0.096	0.065	0.044	0.015	0.390	1.520
1.017	1.893	1.479	0.876	0.456	0.260	0.175	0.133	0.091	0.042	0.487	1.370
0.329	1.976	1.992	1.400	0.922	0.609	0.379	0.247	0.132	0.066	0.585	1.242
0.569	1.593	1.775	1.638	1.297	0.978	0.705	0.501	0.319	0.114	0.682	1.054
0.079	0.792	1.584	2.020	1.901	1.545	0.149	0.792	0.436	0.119	0.780	0.960
0.193	0.677	1.353	1.933	2.417	2.803	0.933	0.967	0.483	0.193	0.877	0.772

 $E_0 = 2.0 \text{ MeV}$ $\psi = 27^\circ$

4.815	0.261	0.109	0.036	0.022	0.022	0.014	0.007	—	—	0.097	1.892
4.040	1.069	0.275	0.138	0.057	0.049	0.032	0.016	0.016	—	0.195	1.757
1.214	2.984	0.965	0.398	0.179	0.119	0.080	0.040	0.020	—	0.292	1.670
0.720	2.304	1.695	0.742	0.421	0.332	0.155	0.122	0.066	0.022	0.390	1.520
0.507	1.673	1.647	1.242	0.849	0.583	0.393	0.241	0.127	0.038	0.487	1.370
0.163	1.142	1.904	1.659	1.170	0.843	0.571	0.354	0.190	0.054	0.585	1.242
0.171	1.111	1.795	1.966	1.667	1.239	0.855	0.470	0.171	0.043	0.692	1.054
0.167	0.500	1.000	1.583	1.917	1.917	1.667	1.083	0.417	0.167	0.780	0.960
0.144	0.289	0.866	1.588	2.165	2.698	2.303	1.010	0.433	0.144	0.845	0.866

$E_0 = 2.0 \text{ Mev}$ $\psi_0 = 45^\circ$

4.509	0.528	0.148	0.055	0.027	0.022	0.016	0.016	0.011	0.011	0.097	1.871
2.699	1.639	0.658	0.237	0.217	0.118	0.079	0.069	0.046	0.007	0.195	1.736
1.046	1.998	1.398	0.669	0.412	0.274	0.214	0.129	0.086	0.043	0.292	1.555
0.688	1.615	1.626	1.102	0.731	0.502	0.317	0.197	0.109	0.033	0.390	1.445
0.539	1.150	1.604	1.533	1.107	0.795	0.539	0.284	0.128	0.043	0.487	1.295
0.228	0.975	1.495	1.690	1.560	1.105	0.747	0.520	0.293	0.098	0.585	1.147
0.263	1.105	1.473	1.631	1.631	1.473	1.263	0.894	0.526	0.158	0.682	0.960
0.194	0.873	1.359	1.747	1.941	1.941	1.747	0.164	0.486	0.097	0.747	0.866

 $E_0 = 2.0 \text{ Mev}$ $\psi_0 = 63^\circ$

3.032	1.476	0.456	0.163	0.137	0.119	0.056	0.013	0.012	—	0.097	1.830
1.128	1.909	1.077	0.772	0.416	0.229	0.178	0.127	0.085	0.017	0.195	1.684
0.510	1.304	1.351	1.221	0.676	0.521	0.403	0.296	0.202	0.095	0.292	1.520
0.489	1.025	1.371	1.293	0.977	0.757	0.599	0.441	0.268	0.079	0.390	1.370
0.283	0.945	1.323	1.394	1.205	1.039	0.850	0.638	0.402	0.118	0.487	1.220
0.200	0.735	1.269	1.537	1.938	1.470	1.069	0.735	0.401	0.134	0.585	1.054
0.103	0.413	1.031	1.856	2.475	1.753	1.238	0.625	0.516	0.206	0.650	0.960

 $E_0 = 2.0 \text{ Mev}$ $\psi_0 = 81^\circ$

1.609	1.757	0.943	0.518	0.277	0.176	0.102	0.055	0.028	—	0.065	1.830
0.794	1.539	1.142	0.831	0.558	0.434	0.360	0.099	0.087	0.037	0.130	1.700
0.357	0.153	1.201	1.185	0.828	0.609	0.390	0.300	0.195	0.032	0.165	1.600
0.126	0.589	1.135	1.219	1.135	1.030	0.567	0.378	0.273	0.126	0.260	1.520
0.139	0.722	1.082	1.249	1.304	1.166	0.833	0.527	0.222	0.056	0.325	1.370
0.083	0.372	0.826	1.197	1.363	1.363	1.074	0.784	0.496	0.165	0.390	1.295
0.062	0.310	0.931	1.615	1.863	1.615	0.993	0.497	0.248	0.062	0.455	1.220

 $E_0 = 1.0 \text{ Mev}$ $\psi_0 = 0$

9.709	0.501	0.131	0.036	0.024	0.024	0.024	0.012	—	—	0.043	0.956
8.995	1.303	0.454	0.117	0.103	0.078	0.052	0.039	0.026	0.013	0.087	0.894
4.865	3.987	0.934	0.531	0.391	0.323	0.253	0.133	0.112	0.042	0.131	0.860
4.285	4.820	1.640	0.937	0.532	0.355	0.167	0.100	0.033	—	0.175	0.780
1.029	4.679	3.650	1.591	0.936	0.534	0.439	0.337	0.225	0.112	0.219	0.740

 $E_0 = 1.0 \text{ Mev}$ $\psi_0 = 27^\circ$

9.409	0.634	0.236	0.107	0.043	0.043	0.032	0.021	—	—	0.043	0.950
7.792	2.023	0.723	0.361	0.156	0.096	0.024	0.024	0.024	0.012	0.087	0.890
2.646	4.852	1.925	0.909	0.788	0.281	0.240	0.107	0.053	0.027	0.131	0.860
2.476	4.477	2.476	1.340	0.933	0.475	0.322	0.220	0.085	0.017	0.175	0.780
2.497	4.184	3.652	2.196	1.017	0.370	0.185	0.115	0.046	0.023	0.219	0.700
0.184	2.664	3.582	2.985	2.112	1.515	1.102	0.735	0.413	0.092	0.262	0.650
0.702	2.667	3.298	3.509	3.158	2.035	1.193	0.631	0.281	0.070	0.306	0.570
0.358	1.521	3.309	3.488	3.130	2.683	2.236	1.521	0.805	0.179	0.335	0.520
0.546	1.637	3.091	4.182	3.819	3.091	2.364	1.637	0.727	0.182	0.365	0.470

 $E_0 = 1.0 \text{ Mev}$ $\psi_0 = 45^\circ$

9.301	0.760	0.274	0.152	0.061	0.061	0.030	—	—	—	0.043	0.940
5.658	3.402	1.072	0.481	0.296	0.259	0.074	0.074	0.037	0.037	0.087	0.878
2.736	4.050	1.948	1.424	0.649	0.401	0.341	0.278	0.232	0.046	0.131	0.820
2.763	3.543	2.643	1.622	1.081	0.721	0.540	0.360	0.180	0.060	0.175	0.740
0.974	3.896	3.328	2.354	1.623	1.055	0.568	0.244	0.162	0.081	0.219	0.700
0.491	1.810	2.962	3.292	3.127	2.804	1.152	0.494	0.329	0.165	0.262	0.620
0.611	1.526	3.358	3.969	3.358	2.441	1.831	1.221	0.611	0.305	0.306	0.520
1.091	1.637	2.728	3.819	4.910	3.273	2.182	1.091	0.545	—	0.335	0.470

 $E_0 = 1.0 \text{ Mev}$ $\psi_0 = 63^\circ$

6.117	2.533	0.736	0.443	0.317	0.241	0.152	0.089	0.025	—	0.043	0.930
2.832	3.714	1.815	1.018	0.848	0.509	0.458	0.220	0.153	0.034	0.087	0.862

0.583	3.385	2.937	1.457	1.188	1.076	0.785	0.381	0.246	0.157	0.101	0.820
0.754	2.805	3.087	2.205	1.730	1.260	0.814	0.473	0.221	0.063	0.175	0.740
0.566	2.803	3.135	3.135	1.959	1.437	1.132	0.827	0.470	0.174	0.219	0.660
0.714	1.939	2.754	3.162	3.060	2.448	1.632	1.020	0.612	0.204	0.262	0.570
0.363	2.355	2.540	2.903	4.717	2.721	1.633	1.088	0.544	0.363	0.292	0.520

$$E_0 = 1.0 \text{ MeV} \quad \psi_0 = 81^\circ$$

2.554	3.538	1.937	1.453	0.528	0.330	0.286	0.176	0.063	0.022	0.043	0.914
1.691	3.026	2.633	1.577	0.553	0.644	0.616	0.364	0.112	0.056	0.073	0.956
0.471	2.138	2.935	2.464	1.630	1.087	0.797	0.543	0.200	0.145	0.102	0.800
1.118	2.032	2.946	3.150	1.727	1.168	0.559	0.406	0.254	0.102	0.131	0.740
0.905	1.810	2.262	2.392	2.327	2.133	1.551	0.646	0.194	0.065	0.160	0.700
0.635	1.515	2.313	2.711	2.632	1.994	1.435	1.037	0.638	0.239	0.189	0.660
0.256	1.024	2.176	3.072	3.594	2.658	1.664	1.024	0.512	0.128	0.219	0.620

$$E_0 = 0.7 \text{ MeV} \quad \psi_0 = 0$$

13.902	0.512	0.211	0.105	0.075	0.030	0.015	0.015	0.015	0.015	0.027	0.672
13.032	1.680	0.391	0.261	0.224	0.083	0.033	0.033	0.016	0.016	0.055	0.636
6.827	6.437	1.463	0.644	0.351	0.234	0.156	0.098	0.058	0.020	0.083	0.614
5.999	6.385	2.465	1.167	0.683	0.430	0.352	0.220	0.110	0.044	0.111	0.560
1.868	6.633	4.467	2.410	1.272	0.912	0.541	0.379	0.271	0.107	0.139	0.533
2.165	5.057	4.871	3.734	2.273	1.258	0.753	0.487	0.271	0.107	0.167	0.475
0.315	3.539	4.954	4.797	3.381	2.559	1.573	1.022	0.629	0.157	0.195	0.440
1.040	2.970	4.306	4.900	4.900	4.610	2.673	1.485	0.594	0.149	0.225	0.370
0.515	2.057	3.860	5.404	5.661	5.661	3.860	1.801	0.772	0.258	0.241	0.335

$$E_0 = 0.7 \text{ MeV} \quad \psi_0 = 27^\circ$$

13.832	0.617	0.347	0.077	0.039	0.039	0.019	—	—	—	0.027	0.664
12.222	2.432	0.602	0.215	0.151	0.108	0.086	0.065	0.043	—	0.055	0.628
3.642	8.013	2.263	1.093	0.442	0.312	0.234	0.156	0.104	0.026	0.083	0.614
3.608	6.124	3.214	1.880	1.122	0.697	0.515	0.364	0.242	0.091	0.111	0.560
3.150	5.813	3.562	2.063	1.650	1.312	1.013	0.675	0.413	0.112	0.139	0.506
0.655	5.243	4.669	3.113	2.294	1.802	1.392	0.983	0.655	0.246	0.167	0.475
1.286	5.273	5.787	4.115	3.086	2.186	1.543	0.900	0.386	0.129	0.195	0.405
1.305	4.473	6.524	4.473	3.169	2.610	1.864	1.305	0.932	0.373	0.223	0.370
0.671	2.012	3.689	6.373	6.037	4.360	3.019	2.012	1.342	0.335	0.232	0.335

$$E_0 = 0.7 \text{ MeV} \quad \psi_0 = 45^\circ$$

12.740	1.389	0.410	0.190	0.142	0.095	0.003	0.003	0.016	0.016	0.027	0.664
8.127	4.522	1.297	0.916	0.381	0.267	0.229	0.191	0.076	0.019	0.055	0.624
3.698	6.233	3.275	1.532	0.845	0.591	0.423	0.264	0.132	0.053	0.083	0.587
3.019	5.129	4.057	2.597	1.623	1.006	0.649	0.422	0.195	0.065	0.111	0.533
0.749	4.543	4.215	3.278	2.435	1.780	1.264	0.843	0.515	0.140	0.139	0.506
0.862	5.170	4.416	3.662	2.909	2.154	1.616	1.077	0.646	0.215	0.167	0.440
0.454	2.424	3.939	4.544	4.393	3.787	2.575	1.515	0.758	0.303	0.186	0.405
0.744	3.223	4.711	4.959	4.463	3.471	2.480	1.736	0.992	0.248	0.204	0.370

$$E_0 = 0.7 \text{ MeV} \quad \psi_0 = 63^\circ$$

9.104	3.711	0.896	0.640	0.347	0.239	0.183	0.146	0.065	0.018	0.027	0.652
4.241	5.417	2.917	1.054	0.556	0.686	0.598	0.294	0.245	0.049	0.055	0.608
2.965	4.604	3.941	2.267	1.465	1.046	0.732	0.458	0.279	0.070	0.083	0.560
1.812	3.111	4.668	3.402	2.042	1.555	1.167	0.826	0.456	0.194	0.111	0.533
0.985	2.955	4.092	4.364	3.519	2.252	1.387	0.774	0.422	0.141	0.139	0.480
0.573	1.719	3.247	4.202	4.202	3.439	2.483	1.528	0.955	0.382	0.167	0.440
0.277	1.387	3.052	4.161	4.489	3.884	3.329	2.497	1.110	0.555	0.186	0.405

$E_c = 0.7 \text{ Mse}$ $\psi = 81^\circ$

4.407	4.684	2.835	1.541	0.832	0.555	0.370	0.216	0.123	0.062	0.027	0.0
3.965	8.860	8.263	2.109	1.353	1.035	0.756	0.597	0.358	0.159	0.046	0.0
1.158	4.135	8.418	2.426	1.874	1.489	1.158	0.772	0.441	0.165	0.065	0.0
0.437	3.644	8.717	2.915	2.332	1.749	1.312	0.948	0.593	0.219	0.083	0.0
0.406	3.347	4.665	3.245	2.332	1.724	1.318	0.913	0.608	0.203	0.102	0.0
1.297	2.593	4.409	4.409	3.242	2.204	1.426	0.778	0.389	0.130	0.120	0.0
1.079	2.518	8.417	3.777	3.597	2.878	2.158	1.439	0.899	0.360	0.139	0.0

 $E_c = 0.4 \text{ Mse}$ $\psi = 0^\circ$

24.444	0.862	0.287	0.104	0.104	0.052	0.026	0.026	—	—	0.012	0.0
22.832	2.493	0.812	0.364	0.280	0.140	0.112	0.084	0.056	—	0.025	0.0
17.723	6.803	1.898	0.716	0.537	0.358	0.215	0.143	0.107	0.072	0.038	0.0
8.400	11.956	4.298	2.149	1.172	0.781	0.503	0.313	0.195	0.078	0.051	0.0
2.291	10.482	8.190	4.328	2.584	1.463	0.829	0.585	0.341	0.098	0.064	0.0
3.345	9.947	6.690	4.930	3.521	2.553	1.760	1.232	0.880	0.352	0.077	0.0
0.903	7.270	8.309	7.530	5.193	3.635	2.467	1.688	0.909	0.260	0.090	0.0
1.479	4.654	6.657	7.366	7.150	5.671	4.191	2.712	1.233	0.498	0.103	0.0
0.891	4.008	7.126	8.907	8.462	6.680	4.454	3.117	1.781	0.445	0.111	0.0

 $E_c = 0.4 \text{ Mse}$ $\psi = 27^\circ$

24.128	1.275	0.319	0.100	0.080	0.060	0.027	0.026	0.026	—	0.012	0.0
19.720	4.661	1.340	0.699	0.379	0.291	0.097	0.058	0.048	0.029	0.025	0.0
10.924	10.738	3.301	1.480	0.835	0.569	0.341	0.190	0.114	0.076	0.038	0.0
4.435	11.621	5.544	3.105	1.863	1.153	0.843	0.709	0.399	0.177	0.061	0.0
3.451	9.951	6.501	4.544	3.624	2.071	1.438	0.690	0.345	0.173	0.064	0.0
0.963	7.708	8.582	6.207	4.495	2.997	2.033	1.284	0.749	0.214	0.077	0.0
0.641	4.009	7.537	8.500	6.254	4.490	3.207	2.085	1.123	0.321	0.090	0.0
0.521	2.604	7.232	9.635	7.813	5.729	3.906	2.344	1.302	0.521	0.098	0.0
0.402	2.817	6.840	10.864	9.657	7.243	4.426	2.414	0.805	0.402	0.107	0.0

 $E_c = 0.4 \text{ Mse}$ $\psi = 45^\circ$

21.838	2.674	0.802	0.443	0.221	0.111	0.083	0.055	0.028	0.028	0.012	0.0
18.355	8.318	2.320	1.425	0.762	0.431	0.331	0.265	0.166	0.099	0.025	0.0
4.486	11.570	5.384	2.645	1.700	1.181	0.803	0.472	0.236	0.094	0.038	0.0
4.532	9.602	5.844	3.817	2.564	1.808	1.372	0.855	0.537	0.179	0.051	0.0
1.597	5.969	8.827	5.633	4.288	2.522	1.933	1.009	0.673	0.336	0.064	0.0
0.640	4.161	5.242	7.042	6.242	4.802	3.041	1.760	0.960	0.320	0.077	0.0
0.285	1.709	4.557	6.551	7.121	6.551	5.412	3.418	1.964	0.570	0.090	0.0
1.119	5.035	8.891	8.391	6.713	5.954	4.475	3.856	2.238	0.559	0.098	0.0

 $E_c = 0.4 \text{ Mse}$ $\psi = 63^\circ$

16.854	6.049	2.016	0.960	0.448	0.256	0.224	0.160	0.096	0.032	0.012	0.0
8.641	7.840	4.763	2.782	1.770	0.969	0.717	0.590	0.253	0.084	0.025	0.0
3.079	8.735	6.787	4.336	2.577	1.697	1.194	0.817	0.440	0.188	0.038	0.0
2.584	6.593	7.662	5.435	3.742	2.673	1.871	1.247	0.713	0.267	0.051	0.0
0.838	4.188	6.282	6.841	6.422	4.487	2.792	1.675	0.838	0.140	0.064	0.0
1.527	5.191	7.939	8.244	6.412	3.969	2.443	1.527	0.610	0.305	0.078	0.0
0.868	3.472	6.944	9.115	8.246	5.642	3.472	2.170	1.302	0.434	0.081	0.0

 $E_c = 0.4 \text{ Mse}$ $\psi = 81^\circ$

6.844	10.348	4.873	1.807	1.533	0.493	0.438	0.328	0.219	—	0.012	0.0
3.748	8.898	6.941	4.164	2.082	0.902	0.764	0.694	0.416	0.139	0.021	0.0
2.330	6.347	8.330	4.660	3.174	2.182	1.388	0.793	0.218	0.099	0.030	0.0
1.967	5.135	6.884	5.900	4.261	3.069	1.967	1.202	0.656	0.219	0.038	0.0
0.796	4.138	7.640	6.525	4.616	3.342	2.547	1.751	1.114	0.318	0.047	0.0
1.818	5.455	7.500	7.045	5.000	3.636	2.500	1.818	1.136	0.455	0.055	0.0
1.211	4.845	6.965	6.663	6.360	5.148	3.634	1.817	1.211	0.606	0.064	0.0

$Z=14$ $E_0 = 6.0 \text{ Mev}$ $\psi = 0^\circ$

1.625	0.122	0.035	0.009	0.005	0.005	0.005	0.004	—	—	0.360	5.537
1.457	0.343	0.101	0.066	0.016	0.010	0.008	0.005	0.002	0.002	0.720	4.989
0.364	1.095	0.353	0.162	0.085	0.047	0.026	0.014	0.007	0.002	1.080	4.648
0.339	0.865	0.551	0.339	0.190	0.100	0.060	0.039	0.019	0.008	1.440	3.986
0.204	0.541	0.669	0.588	0.410	0.237	0.137	0.087	0.062	0.014	1.800	3.400
0.103	0.345	0.560	0.698	0.672	0.457	0.310	0.198	0.103	0.026	2.160	2.882
0.152	0.436	0.626	0.759	0.835	0.759	0.569	0.379	0.190	0.067	2.520	2.101
0.072	0.250	0.536	0.965	1.103	1.001	0.751	0.465	0.215	0.072	2.700	1.840

 $E_0 = 6.0 \text{ Mev}$ $\psi = 27^\circ$

1.647	0.122	0.035	0.007	0.006	0.006	0.004	0.002	0.002	0.001	0.360	5.463
1.359	0.455	0.127	0.069	0.024	0.011	0.009	0.004	0.002	0.002	0.720	4.857
0.370	0.950	0.428	0.220	0.120	0.073	0.048	0.030	0.018	0.005	1.080	4.425
0.357	0.730	0.668	0.364	0.233	0.137	0.078	0.050	0.031	0.012	1.440	3.765
0.208	0.568	0.644	0.555	0.426	0.308	0.211	0.135	0.073	0.017	1.800	3.145
0.082	0.356	0.527	0.609	0.632	0.597	0.433	0.293	0.211	0.047	2.160	2.624
0.065	0.295	0.489	0.652	0.815	0.946	0.750	0.457	0.228	0.065	2.520	2.109

 $E_0 = 6.0 \text{ Mev}$ $\psi = 45^\circ$

1.609	0.172	0.035	0.021	0.011	0.010	0.009	0.006	0.004	0.002	0.360	5.301
0.783	0.735	0.293	0.113	0.079	0.050	0.043	0.027	0.010	0.007	0.720	4.660
0.210	0.619	0.698	0.349	0.190	0.127	0.092	0.051	0.032	0.013	1.080	4.202
0.182	0.577	0.718	0.462	0.320	0.251	0.167	0.111	0.060	0.017	1.440	3.545
0.185	0.496	0.669	0.689	0.519	0.381	0.265	0.173	0.092	0.023	1.800	2.884
0.044	0.288	0.599	0.754	0.799	0.710	0.510	0.310	0.178	0.044	2.160	2.363
0.042	0.209	0.459	0.794	1.003	0.919	0.668	0.418	0.209	0.042	2.340	2.103

 $E_0 = 6.0 \text{ Mev}$ $\psi = 63^\circ$

1.061	0.545	0.191	0.085	0.047	0.035	0.024	0.019	0.005	0.005	0.360	4.984
0.337	0.606	0.437	0.351	0.244	0.108	0.089	0.072	0.046	0.014	0.720	4.345
0.046	0.314	0.436	0.446	0.405	0.339	0.253	0.162	0.086	0.025	1.080	3.987
0.141	0.357	0.474	0.524	0.532	0.449	0.332	0.233	0.141	0.042	1.440	3.109
0.239	0.444	0.547	0.581	0.581	0.547	0.512	0.410	0.273	0.103	1.800	2.360

 $E_0 = 6.0 \text{ Mev}$ $\psi = 81^\circ$

0.273	0.445	0.514	0.261	0.237	0.163	0.098	0.086	0.065	0.001	0.360	4.661
0.094	0.296	0.440	0.490	0.353	0.281	0.245	0.166	0.080	0.043	0.720	4.024
0.037	0.175	0.375	0.500	0.437	0.438	0.350	0.263	0.150	0.050	1.080	3.545
0.081	0.298	0.514	0.595	0.595	0.541	0.487	0.379	0.216	0.054	1.440	2.656

 $E_0 = 4.0 \text{ Mev}$ $\psi = 0^\circ$

2.445	0.130	0.046	0.024	0.011	0.011	0.011	0.008	0.003	0.003	0.243	3.723
2.270	0.396	0.118	0.057	0.033	0.024	0.021	0.012	0.003	—	0.486	3.401
0.550	1.456	0.566	0.226	0.139	0.091	0.058	0.035	0.023	0.010	0.729	3.174
0.734	1.251	0.785	0.380	0.211	0.130	0.086	0.056	0.030	0.013	0.972	2.726
0.307	0.772	0.997	0.851	0.482	0.291	0.195	0.129	0.075	0.033	1.216	2.427
1.188	0.592	0.969	0.915	0.713	0.555	0.444	0.323	0.188	0.054	1.459	2.027
0.185	0.591	0.687	1.035	1.072	0.998	0.739	0.444	0.185	0.037	1.702	1.629

 $E_0 = 4.0 \text{ Mev}$ $\psi = 27^\circ$

2.429	0.176	0.052	0.022	0.017	0.014	0.006	0.003	0.003	—	0.243	3.678
1.997	0.630	0.172	0.088	0.057	0.038	0.022	0.010	0.003	—	0.486	3.327
0.786	1.322	0.532	0.236	0.153	0.089	0.066	0.046	0.031	0.015	0.729	3.029
0.167	1.072	0.976	0.533	0.338	0.238	0.167	0.100	0.062	0.024	0.972	2.721
0.062	0.280	1.041	0.985	0.694	0.403	0.297	0.207	0.123	0.039	1.216	2.423
0.221	0.644	1.047	1.167	0.906	0.641	0.423	0.262	0.141	0.040	1.459	1.827
0.158	0.526	0.999	1.471	1.366	0.999	0.736	0.420	0.263	0.105	1.702	1.420

$E_c = 4.0 \text{ Mev}$ $\psi = 45^\circ$

2.852	0.260	0.068	0.040	0.020	0.014	0.014	0.008	0.003	0.003	0.243	3.596
1.861	0.960	0.338	0.206	0.114	0.068	0.032	0.025	0.021	0.011	0.486	3.195
0.475	1.069	0.712	0.456	0.308	0.204	0.128	0.081	0.043	0.019	0.729	2.879
0.153	0.702	0.829	0.727	0.510	0.357	0.269	0.191	0.115	0.038	0.972	2.578
0.051	0.456	0.742	0.810	0.742	0.624	0.499	0.337	0.203	0.051	1.216	2.220
0.088	0.341	0.644	0.834	0.909	0.834	0.720	0.568	0.417	0.189	1.459	1.820

 $E_c = 4.0 \text{ Mev}$ $\psi = 63^\circ$

1.626	0.743	0.254	0.114	0.077	0.047	0.040	0.020	0.017	0.003	0.243	3.401
0.450	0.981	0.637	0.435	0.303	0.192	0.147	0.081	0.061	0.025	0.486	3.022
0.396	0.661	0.734	0.646	0.492	0.389	0.272	0.176	0.095	0.029	0.729	2.574
0.206	0.489	0.644	0.721	0.708	0.593	0.464	0.322	0.193	0.064	0.972	2.276
0.192	0.461	0.730	0.922	0.922	0.807	0.653	0.461	0.289	0.077	1.216	1.825

 $E_c = 4.0 \text{ Mev}$ $\psi = 81^\circ$

0.593	0.833	0.599	0.455	0.342	0.180	0.126	0.066	0.042	0.006	0.243	3.080
0.817	0.614	0.635	0.571	0.508	0.465	0.296	0.169	0.106	0.042	0.486	2.692
0.063	0.285	0.554	0.728	0.760	0.649	0.507	0.348	0.190	0.047	0.729	2.424
0.152	0.417	0.606	0.758	0.833	0.833	0.682	0.455	0.265	0.076	0.972	1.977

 $E_c = 2.0 \text{ Mev}$ $\psi = 0^\circ$

4.844	0.286	0.092	0.049	0.022	0.006	0.005	0.005	0.005	—	0.120	1.882
3.785	1.225	0.258	0.185	0.154	0.055	0.037	0.025	0.018	0.018	0.240	1.736
1.681	2.462	0.857	0.420	0.303	0.227	0.151	0.092	0.050	0.025	0.360	1.595
0.748	2.243	1.529	0.940	0.589	0.351	0.238	0.159	0.091	0.034	0.480	1.445
0.648	1.426	1.686	1.335	0.959	0.752	0.570	0.389	0.221	0.065	0.600	1.242
0.380	1.186	1.708	1.850	1.565	1.186	0.854	0.474	0.237	0.047	0.720	1.054
0.163	0.814	1.465	2.036	2.035	1.626	1.221	0.732	0.244	0.081	0.780	0.960

 $E_c = 2.0 \text{ Mev}$ $\psi = 27^\circ$

4.655	0.429	0.115	0.044	0.038	0.027	0.017	0.016	0.006	—	0.120	1.870
3.262	1.454	0.469	0.272	0.170	0.102	0.061	0.041	0.020	0.014	0.240	1.705
1.008	2.343	1.296	0.624	0.403	0.269	0.163	0.086	0.048	0.029	0.360	1.595
1.185	1.756	1.386	0.990	0.739	0.554	0.366	0.238	0.066	0.040	0.480	1.370
0.879	1.172	1.608	1.569	1.190	0.879	0.603	0.379	0.207	0.069	0.600	1.242
0.485	1.086	1.521	1.738	1.666	1.304	0.869	0.507	0.290	0.072	0.720	1.054
0.226	0.793	1.472	1.925	1.925	1.585	1.245	0.793	0.340	0.113	0.780	0.960

 $E_c = 2.0 \text{ Mev}$ $\psi = 45^\circ$

3.969	0.846	0.256	0.161	0.060	0.054	0.048	0.024	0.018	—	0.120	1.840
2.108	1.795	0.898	0.433	0.272	0.176	0.104	0.064	0.064	0.024	0.240	1.684
1.166	2.034	1.166	0.785	0.535	0.428	0.226	0.143	0.048	0.047	0.360	1.520
0.559	1.657	1.490	1.154	0.875	0.652	0.466	0.279	0.130	0.037	0.480	1.370
0.225	0.975	1.500	1.525	1.300	1.000	0.725	0.475	0.250	0.075	0.600	1.242
0.872	1.116	1.736	2.232	1.860	1.488	0.868	0.372	0.248	0.124	0.720	0.960

 $E_c = 2.0 \text{ Mev}$ $\psi = 63^\circ$

2.569	1.497	0.691	0.295	0.201	0.173	0.065	0.036	0.029	—	0.120	1.800
1.230	1.665	1.230	0.631	0.486	0.321	0.310	0.124	0.062	0.031	0.240	1.642
0.284	1.246	1.515	1.246	0.899	0.615	0.394	0.237	0.110	0.032	0.360	1.520
0.422	1.183	1.529	1.502	1.186	0.817	0.553	0.243	0.184	0.053	0.480	1.295
0.209	0.670	1.173	1.550	1.591	1.340	1.005	0.712	0.377	0.084	0.600	1.148
0.360	0.841	1.321	1.561	1.681	1.441	1.081	0.721	0.360	0.120	0.660	1.054

 $E_c = 2.0 \text{ Mev}$ $\psi = 81^\circ$

0.181	2.086	1.497	0.771	0.496	0.302	0.197	0.075	0.030	0.015	0.120	1.778
0.405	1.436	1.376	0.870	0.769	0.546	0.405	0.263	0.121	0.020	0.240	1.610
0.184	0.637	0.906	1.174	1.342	1.174	0.805	0.503	0.268	0.101	0.360	1.440
0.222	0.720	1.163	1.440	1.440	1.274	0.942	0.609	0.332	0.065	0.480	1.220
0.246	0.820	1.230	1.476	1.476	1.394	1.230	1.066	0.328	0.082	0.540	1.070

$E = 1.0 \text{ Mev}$ $\psi = 0$

9.495	0.652	0.272	0.085	0.054	0.011	0.011	0.011	—	—	0.054	0.946
6.888	2.399	0.929	0.439	0.322	0.206	0.090	0.036	0.013	—	0.108	0.888
3.347	3.953	1.766	1.194	0.841	0.528	0.286	0.151	0.084	0.034	0.162	0.820
2.273	3.890	2.778	1.617	1.111	0.756	0.505	0.328	0.177	0.076	0.216	0.749
1.006	2.750	3.052	2.616	1.979	1.409	0.973	0.637	0.369	0.134	0.271	0.670
0.627	1.880	3.007	3.383	2.882	2.256	1.629	1.128	0.627	0.125	0.325	0.570
0.730	2.191	3.165	3.651	3.408	2.678	1.704	0.974	0.487	0.243	0.352	0.520

 $E = 1.0 \text{ Mev}$ $\psi = 27^\circ$

9.066	1.059	0.245	0.100	0.067	0.045	0.045	0.011	—	—	0.054	0.940
5.939	2.902	1.220	0.597	0.258	0.244	0.108	0.025	0.014	0.014	0.108	0.878
2.381	4.393	2.254	1.156	0.866	0.559	0.285	0.154	0.058	0.019	0.162	0.820
1.640	3.251	2.968	2.149	1.413	0.905	0.565	0.368	0.198	0.057	0.216	0.749
0.696	2.392	3.264	3.046	2.176	1.525	0.957	0.522	0.261	0.087	0.271	0.670
0.394	1.577	2.563	3.154	3.351	2.257	1.971	0.986	0.394	0.197	0.325	0.570

 $E = 1.0 \text{ Mev}$ $\psi = 45^\circ$

7.444	1.994	0.459	0.314	0.169	0.133	0.085	0.072	0.024	—	0.054	0.953
4.273	3.583	1.742	0.756	0.427	0.230	0.197	0.131	0.066	0.049	0.108	0.872
1.302	3.789	2.771	1.823	1.065	0.616	0.426	0.189	0.142	0.071	0.162	0.820
2.274	3.221	3.060	2.122	1.326	0.905	0.644	0.417	0.227	0.076	0.216	0.700
1.484	3.084	3.321	2.787	1.957	1.423	0.949	0.652	0.256	0.119	0.271	0.620
0.481	2.163	3.365	4.087	3.365	2.404	1.685	0.961	0.481	0.240	0.325	0.520

 $E = 1.0 \text{ Mev}$ $\psi = 63^\circ$

4.913	3.256	1.180	0.642	0.284	0.194	0.129	0.090	0.060	0.015	0.054	0.930
2.268	3.577	1.963	1.331	1.090	0.676	0.349	0.218	0.153	0.044	0.108	0.857
1.611	3.154	2.674	1.851	1.337	0.960	0.651	0.377	0.171	0.034	0.162	0.780

0.725	3.516	2.734	2.121	1.730	1.330	1.004	0.670	0.335	0.112	0.216	0.700
0.422	1.792	3.268	3.373	2.655	1.898	1.370	0.843	0.422	0.106	0.271	0.620

 $E = 1.0 \text{ Mev}$ $\psi = 81^\circ$

3.385	3.789	1.541	0.884	0.531	0.404	0.278	0.101	0.050	0.025	0.054	0.910
1.188	3.248	3.009	1.663	0.911	0.752	0.633	0.317	0.079	0.040	0.108	0.842
0.625	2.189	2.752	2.251	1.564	1.251	1.001	0.688	0.375	0.125	0.162	0.780
0.862	1.847	2.340	2.586	2.340	1.847	1.222	0.736	0.369	0.122	0.216	0.700

 $E = 0.7 \text{ Mev}$ $\psi = 0$

13.444	0.870	0.275	0.122	0.092	0.092	0.046	0.015	0.015	—	0.034	0.668
9.649	3.566	1.231	0.616	0.344	0.163	0.127	0.072	0.054	—	0.069	0.632
5.550	5.550	2.323	1.394	0.800	0.568	0.387	0.258	0.155	0.052	0.104	0.587
2.921	6.404	3.855	1.910	1.423	1.861	0.599	0.412	0.225	0.112	0.138	0.533
0.285	3.325	5.938	3.943	2.518	1.758	1.140	0.665	0.333	0.095	0.173	0.500
0.288	1.918	4.507	5.178	4.411	2.973	1.822	0.959	0.479	0.192	0.208	0.440
0.297	1.190	3.867	5.355	5.355	3.570	2.380	1.487	0.893	0.297	0.225	0.405

 $E = 0.7 \text{ Mev}$ $\psi = 27^\circ$

12.981	1.174	0.453	0.252	0.101	0.034	0.033	0.017	0.016	—	0.034	0.664
8.380	4.317	1.263	1.011	0.421	0.316	0.232	0.126	0.042	0.021	0.069	0.620
3.986	5.606	3.239	1.682	1.028	0.716	0.436	0.218	0.093	0.031	0.104	0.587
2.210	5.061	4.510	2.661	1.714	1.128	0.722	0.451	0.225	0.090	0.138	0.533
1.682	4.485	4.983	4.235	2.554	1.433	0.810	0.436	0.187	0.062	0.173	0.475
0.656	2.404	4.370	5.681	5.026	3.278	1.748	0.874	0.437	0.218	0.208	0.405

 $E = 0.7 \text{ Mev}$ $\psi = 45^\circ$

11.385	2.174	0.719	0.394	0.188	0.103	0.068	0.051	0.034	0.034	0.034	0.660
6.314	4.965	1.922	1.258	0.595	0.412	0.366	0.206	0.092	—	0.069	0.620
1.782	5.987	3.528	2.317	1.390	0.855	0.535	0.356	0.214	0.071	0.104	0.587
1.134	4.194	4.308	2.834	2.154	1.644	1.190	0.794	0.397	0.113	0.138	0.533
0.726	3.226	5.001	4.436	2.581	1.775	1.371	1.049	0.645	0.242	0.173	0.475
0.358	1.789	3.936	5.010	5.010	3.578	2.606	1.431	0.716	0.358	0.208	0.405

$E = 0.7 \text{ M}\mu\text{e}$ $\psi = 63^\circ$

7,967	4,005	1,619	0,746	0,426	0,256	0,170	0,106	0,043	—	0,034	0,625
4,681	4,504	2,728	1,996	1,110	0,698	0,476	0,254	0,127	0,032	0,069	0,604
2,264	4,220	3,705	2,573	1,853	1,338	0,926	0,566	0,309	0,103	0,104	0,560
1,994	3,898	3,989	3,173	2,357	1,722	1,269	0,816	0,453	0,091	0,138	0,506
1,412	3,529	4,800	5,082	3,388	2,117	1,270	0,706	0,282	0,141	0,173	0,440

 $E_c = 0.7 \text{ M}\mu\text{e}$ $\psi_c = 81^\circ$

4,250	5,260	2,996	1,254	0,871	0,348	0,174	0,139	0,105	0,035	0,034	0,648
2,848	4,756	3,881	1,968	1,203	1,093	0,547	0,328	0,164	0,109	0,069	0,592
1,903	3,172	3,807	3,897	3,082	1,450	0,816	0,363	0,181	0,091	0,104	0,533
1,431	3,181	4,294	3,976	3,022	2,067	1,431	0,795	0,477	0,159	0,136	0,480
1,397	3,260	4,192	3,959	3,493	2,562	1,630	0,932	0,466	0,233	0,156	0,452

 $E = 4.0 \text{ M}\mu\text{e}$ $\psi_c = 0^\circ$

23,372	1,654	0,347	0,213	0,134	0,107	0,027	0,027	0,026	—	0,016	0,366
17,957	5,280	1,834	0,917	0,506	0,379	0,221	0,160	0,158	0,032	0,032	0,364
6,222	12,344	4,430	2,240	1,294	0,846	0,498	0,346	0,249	0,100	0,048	0,350
3,685	11,286	6,732	3,827	2,126	1,488	0,992	0,638	0,354	0,112	0,064	0,320
1,076	5,713	10,268	6,855	4,140	2,567	1,490	0,911	0,497	0,166	0,081	0,296
1,080	5,041	8,642	9,382	6,461	3,601	2,160	1,080	0,360	0,360	0,097	0,262
0,548	5,482	10,417	10,965	6,031	3,838	2,193	1,096	0,548	0,548	0,106	0,240

 $E = 0.4 \text{ M}\mu\text{e}$ $\psi_c = 27^\circ$

21,991	2,067	0,909	0,634	0,220	0,165	0,028	0,027	—	—	0,016	0,384
15,328	6,608	2,657	1,022	0,954	0,511	0,238	0,204	0,102	—	0,032	0,362
4,697	11,483	5,927	3,187	1,677	1,062	0,671	0,447	0,224	0,056	0,048	0,340
2,892	7,873	7,632	5,623	3,213	1,767	1,044	0,643	0,402	0,161	0,064	0,320
3,010	5,291	6,294	6,203	5,200	3,922	2,645	1,551	0,821	0,274	0,081	0,284
1,316	4,826	7,019	7,897	7,019	4,387	2,632	1,755	0,877	0,439	0,097	0,262

 $E_c = 0.4 \text{ M}\mu\text{e}$ $\psi_c = 45^\circ$

19,981	3,716	1,279	0,579	0,213	0,183	0,183	0,122	0,061	—	0,016	0,380
11,677	7,963	3,466	1,815	1,031	0,908	0,619	0,248	0,166	0,041	0,032	0,358
6,397	9,453	5,686	3,269	2,132	1,350	0,853	0,426	0,213	0,071	0,048	0,335
1,067	7,039	7,786	5,973	3,626	2,453	1,600	0,960	0,533	0,213	0,064	0,320
1,047	4,320	7,330	7,199	5,498	3,796	2,749	1,833	1,047	0,393	0,081	0,284
1,193	3,578	5,566	7,156	7,554	5,964	3,578	1,988	1,193	0,398	0,089	0,262

 $E_c = 0.4 \text{ M}\mu\text{e}$ $\psi_c = 63^\circ$

14,533	6,390	3,117	1,169	0,545	0,351	0,195	0,078	0,039	0,039	0,016	0,378
8,420	7,409	4,884	2,694	2,245	1,459	0,730	0,505	0,168	0,066	0,032	0,350
2,496	8,486	5,990	4,393	3,195	2,596	1,498	0,898	0,499	0,100	0,048	0,335
1,297	4,446	7,409	7,409	5,187	3,334	1,852	1,112	0,556	0,185	0,064	0,305
0,880	3,228	5,282	5,869	5,869	4,988	3,815	2,924	1,760	0,587	0,081	0,284

 $E_c = 0.4 \text{ M}\mu\text{e}$ $\psi_c = 81^\circ$

11,657	8,401	3,452	1,433	0,977	0,521	0,261	0,130	0,130	0,065	0,016	0,370
5,228	8,888	5,960	3,242	2,196	1,987	0,941	0,314	0,209	0,105	0,032	0,344
3,091	6,181	7,383	5,151	3,434	2,404	1,717	1,202	0,515	0,172	0,048	0,320
0,639	4,789	6,705	6,705	5,428	4,151	2,874	1,916	0,958	0,319	0,064	0,290

 $Z = 22$ $E_c = 6.0 \text{ M}\mu\text{e}$ $\psi_c = 0^\circ$

1,638	0,121	0,043	0,011	0,004	0,002	0,002	0,002	—	—	0,407	5,490
1,237	0,474	0,161	0,070	0,036	0,019	0,004	0,004	0,002	—	0,814	4,980
0,446	0,951	0,419	0,180	0,114	0,079	0,048	0,013	0,008	0,005	1,221	4,420
0,295	0,764	0,666	0,377	0,230	0,144	0,092	0,056	0,030	0,006	1,628	3,760
0,211	0,517	0,681	0,638	0,474	0,293	0,181	0,112	0,060	0,017	2,035	3,140
0,036	0,266	0,621	0,675	0,657	0,639	0,479	0,337	0,089	0,018	2,442	2,620
0,031	0,249	0,499	0,664	0,779	0,748	0,592	0,405	0,218	0,062	2,645	2,360

$E_c = 6.0 \text{ Mev}$ $\psi = 27^\circ$

1.620	0.141	0.045	0.017	0.006	0.005	0.004	0.002	—	—	0.407	5.340
1.061	0.641	0.186	0.078	0.042	0.027	0.020	0.013	0.007	—	0.814	4.820
0.126	0.897	0.539	0.296	0.181	0.089	0.064	0.039	0.022	0.009	1.221	4.420
0.117	0.732	0.641	0.438	0.290	0.196	0.124	0.072	0.038	0.011	1.628	3.760
0.064	0.427	0.748	0.604	0.444	0.337	0.253	0.171	0.102	0.032	2.035	3.140
0.125	0.401	0.602	0.752	0.777	0.627	0.541	0.301	0.150	0.050	2.442	2.860
0.004	0.377	0.680	0.949	0.802	0.707	0.566	0.377	0.236	0.084	2.645	2.100

 $E_c = 6.0 \text{ Mev}$ $\psi = 45^\circ$

1.589	0.200	0.058	0.025	0.011	0.008	0.004	0.002	0.002	—	0.407	5.270
0.710	0.751	0.312	0.156	0.080	0.055	0.043	0.024	0.013	0.003	0.814	4.630
0.104	0.631	0.650	0.387	0.245	0.169	0.111	0.063	0.035	0.010	1.221	4.200
0.079	0.555	0.619	0.499	0.389	0.294	0.205	0.113	0.053	0.016	1.628	3.540
0.053	0.228	0.561	0.666	0.643	0.544	0.403	0.246	0.105	0.018	2.035	2.880
0.027	0.165	0.412	0.604	0.714	0.685	0.577	0.354	0.192	0.055	2.238	2.620

 $E_c = 6.0 \text{ Mev}$ $\psi = 63^\circ$

1.037	0.432	0.233	0.074	0.061	0.047	0.031	0.022	0.015	0.005	0.407	4.980
0.323	0.622	0.461	0.302	0.197	0.143	0.101	0.053	0.046	0.011	0.814	4.370
0.162	0.480	0.588	0.450	0.336	0.246	0.180	0.123	0.072	0.024	1.221	3.760
0.044	0.243	0.452	0.552	0.536	0.508	0.320	0.176	0.088	0.033	1.628	3.320
0.082	0.205	0.328	0.492	0.633	0.821	0.616	0.369	0.164	0.041	2.035	2.620

 $E_c = 6.0 \text{ Mev}$ $\psi = 81^\circ$

0.363	0.478	0.460	0.345	0.233	0.124	0.071	0.053	0.018	0.018	0.407	4.690
0.202	0.365	0.396	0.412	0.459	0.225	0.194	0.132	0.070	0.031	0.814	4.020
0.053	0.237	0.386	0.466	0.493	0.453	0.346	0.227	0.133	0.040	1.221	3.540
0.033	0.198	0.384	0.523	0.628	0.631	0.562	0.331	0.132	0.033	1.628	2.880

 $E_c = 4.0 \text{ Mev}$ $\psi = 0^\circ$

2.420	0.171	0.046	0.019	0.011	0.008	0.006	0.005	0.003	0.003	0.275	3.715
2.159	0.521	0.163	0.072	0.038	0.016	0.013	0.009	0.003	—	0.550	3.340
1.015	1.176	0.513	0.294	0.129	0.074	0.051	0.027	0.020	0.012	0.826	3.020
0.255	1.008	0.849	0.557	0.361	0.233	0.159	0.056	0.053	0.016	1.101	2.720
0.080	0.533	0.830	0.666	0.640	0.440	0.307	0.213	0.127	0.047	1.377	2.420
0.088	0.410	0.674	0.849	0.908	0.791	0.586	0.351	0.205	0.059	1.652	2.020
0.048	0.287	0.573	0.908	1.147	1.051	0.764	0.478	0.191	0.048	1.790	1.820

 $E_c = 4.0 \text{ Mev}$ $\psi = 27^\circ$

2.424	0.207	0.061	0.022	0.011	0.006	0.005	—	—	—	0.275	3.655
1.659	0.540	0.257	0.127	0.065	0.042	0.020	0.010	0.003	0.003	0.550	3.295
0.355	1.269	0.698	0.416	0.199	0.160	0.108	0.060	0.030	0.022	0.826	3.020
0.311	0.907	0.932	0.634	0.435	0.298	0.187	0.112	0.056	0.019	1.101	2.570
0.100	0.536	0.754	0.854	0.754	0.536	0.435	0.255	0.151	0.050	1.377	2.220
0.099	0.396	0.941	1.237	1.040	0.693	0.495	0.345	0.195	0.050	1.652	1.820

 $E_c = 4.0 \text{ Mev}$ $\psi = 45^\circ$

2.440	0.233	0.074	0.043	0.009	0.004	0.006	0.003	—	—	0.275	3.550
0.962	1.100	0.425	0.253	0.169	0.103	0.065	0.042	0.034	—	0.550	3.170
0.216	0.879	0.939	0.475	0.345	0.259	0.178	0.113	0.059	0.022	0.826	2.870
0.166	0.607	0.893	0.823	0.607	0.451	0.304	0.175	0.083	0.018	1.101	2.420
0.080	0.346	0.692	0.852	0.878	0.825	0.612	0.399	0.213	0.053	1.377	2.020
0.103	0.359	0.719	0.976	1.027	0.976	0.719	0.411	0.154	0.051	1.514	1.820

 $E_c = 4.0 \text{ Mev}$ $\psi = 63^\circ$

1.365	0.849	0.333	0.193	0.082	0.032	0.029	0.025	0.025	0.004	0.275	3.400
0.578	0.840	0.650	0.417	0.370	0.220	0.131	0.072	0.054	0.024	0.550	2.980
0.092	0.591	0.767	0.693	0.509	0.397	0.286	0.194	0.111	0.037	0.826	2.720
0.231	0.627	0.825	0.841	0.742	0.577	0.412	0.280	0.148	0.033	1.101	2.120
0.185	0.494	0.926	1.111	0.926	0.679	0.494	0.370	0.247	0.062	1.377	1.820

$E_0 = 4.0 \text{ MeV}$ $\psi = 81^\circ$

0.586	0.899	0.586	0.453	0.233	0.193	0.126	0.087	0.040	0.013	0.275	8.190
0.213	0.509	0.829	0.687	0.474	0.355	0.237	0.142	0.069	0.047	0.550	2.815
0.173	0.480	0.673	0.730	0.653	0.519	0.384	0.288	0.173	0.058	0.826	2.429
0.145	0.387	0.677	0.822	0.870	0.774	0.580	0.435	0.290	0.097	1.101	1.970

 $E_0 = 2.0 \text{ MeV}$ $\psi = 0^\circ$

4.626	0.449	0.114	0.049	0.022	0.016	0.016	0.011	0.011	—	0.135	1.882
3.149	1.356	0.436	0.329	0.148	0.121	0.094	0.054	0.027	0.013	0.271	1.746
1.079	2.298	1.133	0.604	0.410	0.291	0.216	0.140	0.076	0.022	0.407	1.595
1.097	1.970	1.713	0.960	0.600	0.394	0.257	0.171	0.103	0.034	0.542	1.370
1.053	1.816	1.684	1.290	1.000	0.763	0.526	0.342	0.184	0.053	0.678	1.148
0.969	1.696	2.059	1.938	1.332	0.969	0.606	0.494	0.242	0.121	0.814	0.960

 $E_0 = 2.0 \text{ MeV}$ $\psi = 27^\circ$

4.355	0.612	0.135	0.091	0.040	0.034	0.023	0.017	0.006	0.005	0.135	1.870
2.730	1.522	0.650	0.352	0.275	0.153	0.092	0.046	0.030	0.015	0.271	1.705
0.659	2.110	1.343	0.839	0.539	0.360	0.216	0.120	0.060	0.024	0.407	1.595
0.806	1.737	1.551	1.075	0.765	0.538	0.372	0.248	0.145	0.062	0.542	1.370
0.654	1.377	1.687	1.722	1.205	0.826	0.585	0.379	0.207	0.069	0.678	1.148
0.417	1.043	1.355	1.564	1.460	1.251	1.043	0.730	0.417	0.209	0.746	1.054

 $E_0 = 2.0 \text{ MeV}$ $\psi = 45^\circ$

3.786	0.881	0.244	0.212	0.112	0.081	0.038	0.037	0.025	0.019	0.135	1.840
1.871	1.788	0.606	0.547	0.343	0.222	0.194	0.093	0.056	0.018	0.271	1.684
0.273	1.635	1.514	0.984	0.682	0.485	0.348	0.212	0.106	0.030	0.407	1.595
0.395	1.370	1.634	1.502	0.896	0.632	0.422	0.264	0.132	0.053	0.542	1.370
0.570	1.452	1.659	1.400	1.141	0.933	0.726	0.467	0.259	0.104	0.678	1.148

 $E_0 = 2.0 \text{ MeV}$ $\psi = 63^\circ$

2.461	1.723	0.596	0.333	0.183	0.072	0.063	0.040	0.016	0.008	0.135	1.820
1.084	1.822	1.131	0.893	0.464	0.369	0.167	0.060	0.048	0.012	0.271	1.653
0.364	1.304	1.419	1.109	0.825	0.614	0.422	0.269	0.134	0.038	0.407	1.520
0.074	0.633	1.266	1.490	1.341	1.006	0.707	0.447	0.261	0.074	0.542	1.370
0.387	0.968	1.291	1.484	1.420	1.162	0.774	0.452	0.194	0.065	0.610	1.220

 $E_0 = 2.0 \text{ MeV}$ $\psi = 81^\circ$

1.617	1.891	0.959	0.452	0.233	0.205	0.151	0.096	0.041	0.014	0.135	1.767
0.880	1.673	0.202	1.008	0.536	0.386	0.279	0.230	0.129	—	0.271	1.580
0.527	1.338	1.500	1.419	0.973	0.608	0.446	0.284	0.162	0.041	0.407	1.370
0.412	1.153	1.648	1.813	1.813	0.989	0.412	0.247	0.165	0.082	0.542	1.145

 $E_0 = 1.0 \text{ MeV}$ $\psi_0 = 0^\circ$

9.055	0.578	0.285	0.132	0.077	0.044	0.022	0.022	—	—	0.062	0.951
5.139	3.377	1.233	0.426	0.322	0.294	0.176	0.147	0.068	—	0.124	0.858
2.213	4.155	2.849	1.378	0.859	0.542	0.339	0.203	0.113	0.045	0.186	0.820
0.747	3.322	3.322	2.889	1.605	1.008	0.597	0.326	0.149	0.037	0.248	0.740
0.439	1.568	2.571	2.865	2.822	2.132	1.380	0.753	0.313	0.063	0.310	0.670
0.329	1.536	2.962	3.401	2.743	2.085	1.526	0.987	0.439	0.110	0.341	0.620

 $E_0 = 1.0 \text{ MeV}$ $\psi_0 = 27^\circ$

8.573	0.982	0.479	0.274	0.137	0.080	0.023	0.012	0.011	—	0.062	0.946
5.128	3.123	1.378	0.643	0.429	0.306	0.199	0.122	0.046	0.015	0.124	0.878
1.503	3.873	2.550	1.491	1.034	0.746	0.457	0.264	0.144	0.072	0.186	0.820
0.777	2.979	3.162	2.245	1.641	1.166	0.620	0.475	0.216	0.043	0.248	0.740
1.459	2.756	2.918	2.594	2.188	1.702	1.216	0.811	0.405	0.081	0.310	0.620
1.300	3.119	3.639	3.119	2.839	1.819	1.170	0.650	0.260	0.130	0.341	0.570

 $E_0 = 1.0 \text{ MeV}$ $\psi_0 = 45^\circ$

7.629	1.714	0.563	0.340	0.235	0.066	0.052	0.026	0.013	—	0.062	0.940
3.302	3.536	1.787	0.970	0.665	0.323	0.190	0.133	0.076	0.038	0.124	0.865
0.686	3.618	3.306	1.684	1.092	0.749	0.530	0.312	0.156	0.062	0.186	0.820

0,808	1,796	3,288	3,237	1,850	1,285	0,873	0,566	0,257	0,061	0,248	0,740
0,346	1,613	2,880	3,341	2,765	2,189	1,498	0,921	0,461	0,115	0,310	0,620

 $E_0 = 1,0 \text{ Mэ}$ $\psi = 63^\circ$

5,178	3,100	1,138	0,627	0,462	0,132	0,082	0,033	—	—	0,062	0,930
2,706	3,343	1,914	1,658	0,893	0,587	0,255	0,204	0,128	0,061	0,124	0,852
1,178	2,825	2,912	2,180	1,521	1,000	0,652	0,391	0,174	0,043	0,186	0,780
0,458	2,106	3,114	2,747	1,923	1,465	1,099	0,824	0,458	0,091	0,248	0,700
0,489	1,466	3,421	3,584	2,281	1,629	1,140	0,652	0,326	0,163	0,279	0,660

 $E_0 = 1,0 \text{ Mэ}$ $\psi = 81^\circ$

3,854	3,634	1,734	0,661	0,551	0,275	0,110	0,110	—	—	0,062	0,915
2,494	2,857	2,403	1,859	1,133	0,680	0,408	0,181	0,091	—	0,124	0,826
1,859	2,719	3,119	2,559	1,679	0,960	0,560	0,320	0,160	0,080	0,186	0,740
1,850	2,700	3,300	3,000	2,250	1,350	0,600	0,300	0,150	0,150	0,248	0,660

 $E_0 = 0,7 \text{ Mэ}$ $\psi = 0^\circ$

13,162	1,110	0,365	0,159	0,079	0,063	0,032	—	—	—	0,039	0,668
8,811	3,767	1,596	0,702	0,532	0,277	0,170	0,064	0,064	0,043	0,079	0,624
2,766	5,701	3,542	2,024	1,181	0,742	0,506	0,337	0,169	0,067	0,119	0,587
1,663	5,189	4,191	3,060	2,329	0,998	0,665	0,333	0,200	0,133	0,155	0,533
1,245	3,509	4,980	3,735	2,490	1,924	1,358	1,019	0,566	0,266	0,198	0,475
0,758	2,652	4,545	5,492	3,788	2,273	1,515	0,947	0,568	0,189	0,218	0,440

 $E_0 = 0,7 \text{ Mэ}$ $\psi = 27^\circ$

12,559	1,651	0,350	0,200	0,183	0,100	0,017	—	—	—	0,039	0,664
7,795	4,201	1,864	0,988	0,584	0,427	0,157	0,067	0,045	—	0,079	0,620

2,766	5,457	3,577	2,028	1,290	0,811	0,516	0,332	0,184	0,074	0,119	0,587
1,575	4,440	4,655	3,509	1,790	1,217	0,788	0,501	0,215	0,072	0,158	0,533
0,814	3,140	4,769	4,804	3,140	2,210	1,396	0,814	0,349	0,116	0,198	0,475

 $E_0 = 0,7 \text{ Mэ}$ $\psi_0 = 45^\circ$

10,465	2,964	0,908	0,352	0,130	0,111	0,093	0,074	0,037	0,019	0,039	0,660
6,488	4,506	2,460	1,244	0,636	0,442	0,304	0,221	0,111	0,055	0,079	0,608
3,706	4,813	3,869	2,310	1,444	0,963	0,578	0,385	0,193	0,086	0,119	0,560
2,154	4,215	4,402	3,278	2,342	1,499	0,937	0,562	0,281	0,094	0,158	0,506
0,778	2,867	4,635	4,066	2,897	2,125	1,545	1,159	0,773	0,193	0,198	0,475

 $E_0 = 0,7 \text{ Mэ}$ $\psi_0 = 63^\circ$

8,148	3,978	1,582	0,647	0,407	0,192	0,192	0,130	0,048	0,024	0,039	0,652
4,212	5,018	2,857	1,575	1,026	0,806	0,586	0,193	0,183	0,110	0,079	0,604
2,048	3,968	4,608	2,816	1,728	1,216	0,768	0,448	0,192	0,064	0,119	0,560
1,367	3,977	4,226	2,983	2,237	1,740	1,367	0,994	0,621	0,249	0,158	0,506
1,011	3,489	4,652	4,045	2,832	2,023	1,416	0,809	0,405	0,202	0,178	0,480

 $E_0 = 0,7 \text{ Mэ}$ $\psi_0 = 81^\circ$

5,258	5,711	2,080	1,210	0,643	0,340	0,113	0,076	—	—	0,039	0,648
2,206	5,699	3,615	1,838	1,409	0,930	0,430	0,369	0,061	—	0,079	0,600
0,749	3,422	4,063	3,748	2,352	1,497	0,962	0,642	0,320	0,107	0,119	0,560
0,494	2,470	5,188	4,694	2,470	1,723	1,235	0,741	0,494	0,247	0,158	0,506

 $E_0 = 0,4 \text{ Mэ}$ $\psi_0 = 0^\circ$

22,510	2,186	0,645	0,224	0,168	0,112	0,084	0,056	0,028	0,028	0,018	0,384
14,000	7,067	2,793	1,283	0,868	0,566	0,368	0,113	0,113	—	0,037	0,364
8,673	11,020	5,714	3,265	2,177	1,361	0,748	0,340	0,204	0,068	0,055	0,350
1,670	6,660	8,349	5,725	3,459	2,266	1,551	0,954	0,477	0,239	0,074	0,320
2,280	6,402	8,661	6,967	4,331	2,824	1,883	1,130	0,565	0,188	0,093	0,284
2,482	5,586	7,137	7,753	6,827	4,084	2,482	0,931	0,621	0,310	0,162	0,262

$E_0 = 0.4 \text{ Mse}$ $\psi_0 = 27^\circ$

20,927	2,792	1,234	0,412	0,263	0,206	0,117	0,059	0,029	—	0,018	0,364
13,270	6,511	3,086	1,566	1,043	0,417	0,334	0,209	0,063	0,063	0,037	0,362
3,077	11,255	5,348	3,297	2,198	1,465	0,679	0,513	0,293	0,147	0,055	0,350
1,582	6,988	8,307	5,670	3,560	2,242	1,450	0,791	0,366	0,182	0,074	0,320
1,956	5,282	7,238	7,238	5,086	3,325	2,347	1,565	0,762	0,391	0,063	0,264

 $E_0 = 0.4 \text{ Mse}$ $\psi_0 = 45^\circ$

18,279	5,029	1,677	0,516	0,387	0,161	0,067	0,032	—	—	0,018	0,382
9,741	8,499	4,584	2,101	1,480	0,907	0,219	0,191	0,191	—	0,037	0,358
4,390	8,604	5,707	3,657	2,810	2,019	1,317	0,760	0,351	0,175	0,055	0,335
2,759	8,278	7,142	5,519	3,571	2,272	1,298	0,974	0,487	0,487	0,074	0,305
2,181	5,453	7,088	7,361	6,543	4,635	2,726	1,362	0,545	0,273	0,063	0,262

 $E_0 = 0.4 \text{ Mse}$ $\psi_0 = 63^\circ$

13,860	7,111	2,582	1,449	0,725	0,181	0,136	0,136	0,091	0,045	0,018	0,360
8,589	8,186	4,160	2,952	1,610	1,342	0,671	0,470	0,201	0,067	0,037	0,354
3,025	6,838	7,627	5,260	2,893	1,841	1,184	0,657	0,395	0,131	0,055	0,335
2,223	5,279	5,835	6,891	5,279	3,334	2,501	1,111	0,556	0,278	0,074	0,305
2,053	4,926	6,158	6,158	5,337	3,695	2,873	1,642	1,232	0,410	0,083	0,260

 $E_0 = 0.4 \text{ Mse}$ $\psi_0 = 81^\circ$

11,813	8,064	3,466	1,961	0,637	0,354	0,141	0,141	0,071	0,071	0,018	0,374
5,585	8,066	5,693	3,222	1,933	1,933	1,611	0,480	0,108	—	0,037	0,350
1,571	4,713	6,509	6,509	4,713	2,693	1,571	0,898	0,449	0,224	0,055	0,335
3,678	5,977	5,517	5,057	4,598	3,678	2,759	1,839	0,820	0,460	0,074	0,290

 $Z = 32$ $E_0 = 6.0 \text{ Mse}$ $\psi_0 = 0^\circ$

1,650	0,119	0,032	0,020	0,006	0,004	0,004	0,004	0,002	0,002	0,442	5,430
1,278	0,482	0,154	0,073	0,038	0,029	0,018	0,011	0,002	0,002	0,885	4,796
0,452	0,903	0,452	0,214	0,185	0,096	0,066	0,039	0,018	0,006	1,327	4,207
0,228	0,730	0,712	0,434	0,292	0,192	0,119	0,073	0,036	0,009	1,770	3,543
0,163	0,489	0,717	0,734	0,554	0,359	0,228	0,130	0,065	0,033	2,213	2,881
0,168	0,504	0,672	0,728	0,784	0,728	0,560	0,336	0,168	0,112	2,655	2,100

 $E_0 = 6.0 \text{ Mse}$ $\psi_0 = 27^\circ$

1,615	0,162	0,038	0,019	0,009	0,009	0,008	0,001	0,002	—	0,442	5,362
0,980	0,618	0,225	0,137	0,064	0,039	0,032	0,022	0,010	0,005	0,885	4,690
0,099	0,687	0,561	0,373	0,248	0,165	0,116	0,076	0,043	0,013	1,327	4,205
0,080	0,502	0,708	0,525	0,371	0,263	0,183	0,114	0,068	0,017	1,770	3,548
0,093	0,824	0,556	0,648	0,579	0,483	0,370	0,231	0,139	0,046	2,213	2,889
0,210	0,461	0,597	0,629	0,629	0,587	0,503	0,336	0,210	0,084	2,434	2,361

 $E_0 = 6.0 \text{ Mse}$ $\psi_0 = 45^\circ$

1,422	0,268	0,105	0,045	0,031	0,017	0,012	0,010	0,008	0,002	0,442	5,212
0,526	0,659	0,424	0,211	0,129	0,108	0,072	0,030	0,027	0,018	0,885	4,533
0,178	0,517	0,532	0,470	0,301	0,192	0,127	0,085	0,047	0,014	1,327	3,583
0,068	0,418	0,622	0,534	0,437	0,350	0,262	0,185	0,107	0,029	1,770	3,326
0,065	0,273	0,654	0,818	0,763	0,491	0,327	0,218	0,163	0,055	2,213	2,627

 $E_0 = 6.0 \text{ Mse}$ $\psi_0 = 63^\circ$

0,861	0,533	0,242	0,161	0,109	0,062	0,031	0,029	0,013	0,003	0,442	4,920
0,457	0,535	0,426	0,334	0,202	0,158	0,127	0,088	0,061	0,005	0,885	4,183
0,207	0,513	0,555	0,466	0,355	0,274	0,207	0,141	0,081	0,022	1,327	3,547
0,121	0,328	0,466	0,618	0,500	0,448	0,360	0,259	0,155	0,052	1,770	3,106
0,086	0,289	0,578	0,759	0,723	0,578	0,398	0,253	0,108	0,036	1,991	2,664

$E = 6.0 \text{ Mev}$ $\psi = 81^\circ$

0.378	0.560	0.428	0.281	0.186	0.152	0.081	0.038	0.019	0.014	0.442	4.632
0.124	0.448	0.416	0.382	0.362	0.315	0.191	0.124	0.048	0.010	0.885	4.601
0.089	0.285	0.448	0.552	0.552	0.416	0.303	0.196	0.107	0.056	1.357	3.326
0.118	0.353	0.610	0.647	0.647	0.500	0.353	0.206	0.088	0.029	1.546	2.889

 $E = 4.0 \text{ Mev}$ $\psi = 81^\circ$

2.306	0.181	0.069	0.022	0.019	0.005	0.005	0.003	0.003	0.003	0.300	3.709
1.835	0.716	0.189	0.104	0.065	0.046	0.039	0.013	0.003	0.003	0.600	3.323
0.364	1.294	0.668	0.392	0.252	0.117	0.107	0.089	0.114	0.014	0.900	3.021
0.206	0.929	0.985	0.596	0.429	0.318	0.222	0.127	0.063	0.016	1.200	2.571
0.051	0.384	0.870	0.896	0.742	0.589	0.435	0.307	0.179	0.051	1.500	2.224
0.088	0.221	0.442	0.575	0.751	0.884	0.793	0.751	0.398	0.044	1.650	2.025

 $E = 4.0 \text{ Mev}$ $\psi = 27^\circ$

2.358	0.221	0.083	0.028	0.017	0.008	0.006	0.006	0.005	0.005	0.300	3.655
1.356	0.944	0.303	0.180	0.102	0.081	0.049	0.028	0.011	—	0.600	3.273
0.110	1.060	0.857	0.511	0.269	0.198	0.154	0.110	0.028	0.016	0.900	3.022
0.124	0.610	0.792	0.792	0.572	0.410	0.286	0.181	0.095	0.029	1.200	2.579
0.036	0.322	0.572	0.715	0.822	0.822	0.643	0.358	0.179	0.036	1.500	2.220
0.194	0.517	0.711	0.840	0.840	0.776	0.646	0.517	0.323	0.129	1.650	1.821

 $E = 4.0 \text{ Mev}$ $\psi = 45^\circ$

2.230	0.381	0.100	0.050	0.028	0.018	0.006	0.003	—	—	0.300	3.552
0.867	1.014	0.574	0.285	0.185	0.134	0.065	0.047	0.026	0.009	0.600	3.156
0.166	0.788	0.823	0.567	0.415	0.297	0.207	0.131	0.069	0.021	0.900	2.877
0.192	0.590	0.851	0.783	0.563	0.439	0.330	0.220	0.123	0.041	1.200	2.424
0.144	0.385	0.577	0.769	0.865	0.961	0.673	0.385	0.144	0.048	1.500	2.020

 $E = 4.0 \text{ Mev}$ $\psi = 63^\circ$

1.342	0.837	0.336	0.176	0.122	0.057	0.034	0.027	0.023	0.004	0.300	3.383
0.569	0.820	0.674	0.456	0.311	0.225	0.165	0.099	0.053	0.007	0.600	2.962
0.293	0.574	0.715	0.748	0.596	0.423	0.282	0.163	0.076	0.022	0.900	2.571
0.181	0.544	0.847	1.028	0.847	0.575	0.363	0.212	0.091	0.030	1.200	2.129

 $E = 4.0 \text{ Mev}$ $\psi = 81^\circ$

0.654	0.930	0.691	0.297	0.222	0.148	0.115	0.050	0.030	0.013	0.300	3.175
0.347	0.591	0.771	0.784	0.347	0.295	0.270	0.103	0.064	0.039	0.600	2.778
0.252	0.554	0.680	0.730	0.730	0.604	0.403	0.252	0.151	0.050	0.900	2.276
0.534	0.846	0.846	0.801	0.668	0.534	0.401	0.267	0.134	0.045	1.050	1.973

 $E = 2.0 \text{ Mev}$ $\psi = 0^\circ$

4.468	0.586	0.097	0.051	0.051	0.040	0.023	0.017	0.006	0.005	0.148	1.871
2.593	1.711	0.626	0.372	0.198	0.119	0.103	0.079	0.024	0.016	0.297	1.715
0.509	2.264	1.557	0.877	0.439	0.255	0.170	0.113	0.057	0.028	0.445	1.695
0.417	1.356	1.642	1.408	0.912	0.652	0.469	0.287	0.130	0.026	0.594	1.870
0.487	1.190	1.623	1.840	1.569	1.028	0.541	0.271	0.108	0.064	0.743	1.148
0.333	0.999	1.831	1.997	1.831	1.332	0.666	0.166	0.166	0.166	0.817	1.064

 $E = 2.0 \text{ Mev}$ $\psi = 27^\circ$

4.117	0.758	0.229	0.094	0.094	0.029	0.029	0.018	0.006	—	0.148	1.861
2.509	1.579	0.754	0.456	0.263	0.140	0.096	0.088	0.044	0.009	0.297	1.681
1.067	1.767	1.436	0.820	0.458	0.410	0.252	0.205	0.158	0.016	0.445	1.520
0.428	1.313	1.649	1.466	1.038	0.672	0.397	0.214	0.092	0.031	0.594	1.370
0.494	1.050	1.421	1.668	1.606	1.112	0.741	0.371	0.185	0.062	0.743	1.148

 $E = 2.0 \text{ Mev}$ $\psi = 45^\circ$

3.509	1.065	0.396	0.253	0.102	0.061	0.034	0.007	0.007	—	0.148	1.840
1.642	1.752	0.893	0.639	0.375	0.265	0.187	0.176	0.033	0.011	0.297	1.674
0.526	1.401	1.421	0.993	0.837	0.584	0.370	0.350	0.078	0.019	0.445	1.520

0,545	0,999	1,181	1,317	1,317	1,136	0,681	0,363	0,136	0,045	0,594	1,295
0,318	0,796	1,194	1,432	1,512	1,422	0,875	0,398	0,159	0,080	0,668	1,229

 $E = 2.0 \text{ Mev} \quad \psi_c = 63^\circ$

2,446	1,622	0,627	0,448	0,197	0,061	0,045	0,045	0,018	—	0,148	1,809
1,224	1,606	1,163	0,673	0,597	0,413	0,229	0,138	0,061	0,061	0,297	1,622
0,612	1,133	1,194	1,011	0,857	0,735	0,582	0,429	0,275	0,092	0,445	1,445
0,309	0,849	1,313	1,467	1,390	1,158	0,695	0,309	0,155	0,077	0,594	1,295

 $E_c = 2.0 \text{ Mev} \quad \psi_c = 81^\circ$

1,889	1,875	0,611	0,486	0,236	0,236	0,181	0,069	0,028	0,014	0,148	1,778
1,031	1,635	1,560	0,881	0,478	0,377	0,201	0,075	0,050	—	0,297	1,590
0,310	0,826	1,188	1,291	1,291	0,930	0,568	0,310	0,155	0,052	0,445	1,445
0,581	0,926	1,162	1,245	1,329	1,162	0,664	0,332	0,166	0,083	0,520	1,295

 $E_c = 1.0 \text{ Mev} \quad \psi_c = 0^\circ$

8,529	1,103	0,399	0,223	0,200	0,050	0,035	0,023	—	—	0,068	0,946
5,086	2,808	1,430	0,883	0,442	0,353	0,212	0,124	0,035	0,018	0,136	0,878
0,993	3,848	3,103	1,824	0,962	0,528	0,372	0,248	0,156	0,062	0,204	0,820
0,468	1,940	3,278	2,810	1,940	1,358	0,870	0,468	0,268	0,134	0,272	0,740
0,914	2,351	3,004	3,135	2,351	1,567	0,914	0,522	0,261	0,131	0,306	0,660

 $E_c = 1.0 \text{ Mev} \quad \psi_c = 27^\circ$

7,775	1,592	0,564	0,301	0,100	0,063	0,063	0,050	0,050	0,013	0,068	0,946
4,483	3,160	1,580	0,830	0,810	0,257	0,158	0,099	0,099	0,020	0,136	0,873
0,924	3,363	2,698	1,700	1,220	0,887	0,628	0,443	0,259	0,074	0,204	0,820
0,711	2,134	3,467	2,578	1,778	1,245	0,800	0,445	0,267	0,089	0,272	0,740
0,964	2,066	2,617	2,755	2,617	1,791	1,102	0,689	0,413	0,138	0,306	0,660

 $E_c = 1.0 \text{ Mev} \quad \psi_c = 45^\circ$

6,849	1,922	0,737	0,448	0,332	0,207	0,116	0,043	—	—	0,068	0,938
2,891	3,394	1,860	1,131	0,751	0,603	0,478	0,402	0,101	—	0,136	0,861
0,503	2,711	3,219	2,033	1,525	1,150	0,791	0,508	0,282	0,113	0,204	0,780
1,124	2,247	2,729	2,729	2,247	1,445	0,802	0,482	0,321	0,161	0,272	0,700

 $E_c = 1.0 \text{ Mev} \quad \psi_c = 63^\circ$

5,635	2,901	0,961	0,591	0,333	0,185	0,092	0,037	0,018	—	0,068	0,930
3,132	3,852	1,722	1,065	0,877	0,658	0,251	0,219	0,031	—	0,136	0,847
1,227	2,515	3,497	1,963	1,472	1,043	0,429	0,307	0,245	0,123	0,204	0,770
0,571	1,571	2,143	2,571	2,714	2,429	1,286	0,571	0,286	0,143	0,272	0,700

 $E_c = 1.0 \text{ Mev} \quad \psi_c = 81^\circ$

4,663	3,297	1,396	0,713	0,445	0,148	0,148	0,060	—	—	0,068	0,920
2,986	3,909	2,389	1,194	0,814	0,543	0,109	0,108	0,054	—	0,136	0,826
1,219	2,743	3,455	2,540	1,321	0,914	0,610	0,406	0,203	0,102	0,204	0,740
1,124	2,729	3,371	2,889	1,926	1,124	0,642	0,321	0,161	—	0,238	0,700

 $E_c = 0.7 \text{ Mev} \quad \psi_c = 0^\circ$

12,114	1,529	0,789	0,296	0,101	0,101	0,067	0,034	0,017	—	0,043	0,668
6,773	4,320	1,764	1,099	0,843	0,511	0,256	0,255	0,077	0,026	0,087	0,628
1,718	5,483	4,063	2,350	1,469	0,930	0,538	0,294	0,147	0,049	0,181	0,587
0,915	4,576	3,890	3,546	2,402	1,602	0,915	0,572	0,229	0,114	0,175	0,480
1,291	4,978	4,978	3,184	2,212	1,659	1,291	0,737	0,369	0,184	0,197	0,480

 $E_c = 0.7 \text{ Mev} \quad \psi_c = 27^\circ$

11,514	2,059	0,680	0,304	0,161	0,107	0,090	0,054	—	—	0,043	0,668
5,599	4,542	2,628	1,143	1,028	0,371	0,314	0,200	0,114	0,086	0,087	0,624
1,538	3,703	3,988	3,134	1,994	1,263	0,798	0,399	0,171	0,057	0,131	0,587
1,279	3,127	4,122	3,263	2,558	2,132	0,995	0,711	0,426	0,142	0,175	0,533
1,894	3,314	4,025	3,551	2,601	1,894	1,420	0,947	0,710	0,473	0,197	0,480

$E_0 = 0.7 \text{ Mэв}$ $\psi = 45^\circ$											
9.363	3.330	0.964	0.607	0.293	0.293	0.105	0.084	0.021	—	0.043	0.664
4.504	4.888	2.339	1.746	1.047	0.803	0.628	0.279	0.070	0.035	0.087	0.612
2.214	4.427	3.985	2.804	1.845	1.107	0.738	0.443	0.221	0.074	0.131	0.560
1.098	3.111	3.843	3.660	3.111	2.196	1.464	0.732	0.366	0.183	0.175	0.506

$E_0 = 0.7 \text{ Mэв}$ $\psi = 63^\circ$											
7.609	3.805	1.760	0.061	0.440	0.285	0.233	0.026	0.026	—	0.043	0.656
2.934	5.023	3.156	2.089	1.289	0.933	0.687	0.178	0.989	0.089	0.087	0.608
1.342	3.097	4.129	3.509	2.271	1.342	0.929	0.619	0.413	0.207	0.131	0.560
1.581	3.162	3.953	3.953	3.162	2.055	1.107	0.474	0.158	0.158	0.153	0.506

$E_0 = 0.7 \text{ Mэв}$ $\psi = 81^\circ$											
6.508	4.985	1.892	1.108	0.785	0.369	0.231	0.046	—	—	0.043	0.628
3.336	4.710	2.845	2.257	1.570	1.177	0.981	0.589	0.294	0.098	0.087	0.560
1.443	3.207	4.650	3.207	2.405	1.604	0.962	0.641	0.481	0.160	0.109	0.533
0.780	2.600	4.941	4.421	2.860	1.560	1.040	0.780	0.520	0.260	0.131	0.506

$E_0 = 0.4 \text{ Mэв}$ $\psi_0 = 0^\circ$											
21.071	2.520	0.967	0.523	0.352	0.205	0.088	0.088	0.059	0.029	0.020	0.386
11.296	7.856	3.533	1.580	1.441	0.883	0.465	0.232	0.186	—	0.041	0.364
2.078	9.351	6.961	3.948	2.286	1.558	1.039	0.727	0.416	0.208	0.061	0.350
2.505	7.058	6.831	6.148	3.871	2.732	1.821	1.138	0.455	0.223	0.082	0.305
1.124	5.247	8.621	7.871	4.498	2.999	1.874	1.124	0.750	0.375	0.092	0.290

$E_0 = 0.4 \text{ Mэв}$ $\psi_0 = 27^\circ$											
19.745	3.449	1.202	0.886	0.380	0.158	0.127	0.063	0.032	—	0.020	0.384
10.813	7.986	3.224	2.460	1.538	0.843	0.446	0.248	0.149	0.050	0.041	0.360
6.085	7.233	5.970	4.248	2.755	1.607	0.918	0.574	0.345	0.115	0.061	0.335
2.439	4.335	5.690	5.961	5.148	3.794	2.439	1.626	1.034	0.271	0.082	0.305
2.122	5.305	7.427	6.366	4.775	3.713	2.653	1.591	0.531	—	0.092	0.290

$E_0 = 0.4 \text{ Mэв}$ $\psi_0 = 45^\circ$											
17.294	4.842	1.857	0.933	0.546	0.255	0.218	0.146	0.036	—	0.020	0.382
9.125	7.138	5.090	2.607	1.924	0.807	0.621	0.559	0.062	—	0.041	0.358
4.837	7.463	6.219	4.422	2.902	1.797	1.105	0.691	0.276	0.138	0.061	0.335
2.495	6.415	7.128	6.415	4.277	2.495	1.425	1.069	0.713	0.356	0.032	0.305

$E_0 = 0.4 \text{ Mэв}$ $\psi_0 = 63^\circ$											
14.788	5.970	2.939	1.148	0.689	0.276	0.275	0.184	0.046	—	0.020	0.380
9.052	7.413	4.292	3.043	1.327	1.326	0.780	0.468	0.390	0.156	0.041	0.354
2.410	6.489	7.602	5.933	3.337	1.854	1.113	0.556	0.371	0.185	0.061	0.335
3.448	8.867	6.897	4.433	3.448	2.463	1.970	1.478	0.985	0.493	0.082	0.290

$E_0 = 0.4 \text{ Mэв}$ $\psi_0 = 81^\circ$											
13.411	6.931	2.411	1.884	1.055	0.377	0.226	0.226	0.075	—	0.020	0.376
5.824	8.807	5.898	2.557	2.131	1.420	0.852	0.568	0.568	0.284	0.041	0.352
2.543	5.339	6.611	6.356	4.068	2.288	1.271	0.509	0.254	—	0.051	0.342
3.101	5.427	5.815	4.652	3.877	3.101	2.326	1.163	0.388	—	0.061	0.330

Z=50

$E_0 = 6.0 \text{ Mэв}$ $\psi_0 = 0^\circ$											
1.637	0.149	0.041	0.010	0.009	0.008	0.007	0.007	0.004	0.004	0.492	5.332
0.951	0.560	0.320	0.116	0.068	0.047	0.047	0.029	0.016	0.005	0.984	4.631
0.233	0.734	0.561	0.341	0.242	0.173	0.117	0.065	0.034	0.013	1.476	3.980

0,076	0,326	0,522	0,587	0,533	0,380	0,272	0,185	0,098	0,033	1,968	3,329
0,059	0,316	0,493	0,572	0,572	0,533	0,434	0,296	0,138	0,059	2,214	2,888

 $E_0 = 6.0 \text{ MeV} \quad \psi_0 = 27^\circ$

1,627	0,151	0,050	0,039	0,013	0,006	0,004	0,004	0,002	0,002	0,492	5,273
0,775	0,673	0,301	0,179	0,104	0,095	0,042	0,033	0,015	0,006	0,984	4,506
0,102	0,510	0,714	0,397	0,220	0,209	0,143	0,092	0,051	0,015	1,476	3,985
0,093	0,449	0,729	0,645	0,449	0,323	0,238	0,163	0,048	0,028	1,968	3,104
0,028	0,332	0,802	0,691	0,553	0,470	0,387	0,276	0,166	0,055	2,214	2,660

 $E_0 = 6.0 \text{ MeV} \quad \psi_0 = 45^\circ$

1,270	0,412	0,144	0,052	0,034	0,020	0,020	0,011	0,002	0,002	0,492	5,080
0,504	0,647	0,403	0,208	0,197	0,135	0,069	0,053	0,037	0,007	0,984	4,401
0,054	0,487	0,582	0,487	0,359	0,277	0,196	0,129	0,031	0,027	1,476	3,762
0,118	0,290	0,402	0,543	0,685	0,638	0,425	0,236	0,118	0,047	1,968	2,887

 $E_0 = 6.0 \text{ MeV} \quad \psi_0 = 63^\circ$

0,792	0,503	0,273	0,163	0,107	0,066	0,047	0,013	0,010	0,006	0,492	5,057
0,357	0,500	0,455	0,435	0,189	0,163	0,130	0,053	0,033	—	0,984	4,813
0,202	0,476	0,519	0,447	0,875	0,303	0,231	0,158	0,086	0,029	1,476	3,549
0,050	0,324	0,498	0,548	0,523	0,423	0,299	0,199	0,124	0,025	1,722	3,327

 $E_0 = 6.0 \text{ MeV} \quad \psi_0 = 81^\circ$

0,380	0,537	0,481	0,329	0,177	0,101	0,071	0,041	0,025	0,005	0,492	4,533
0,290	0,453	0,557	0,488	0,343	0,256	0,151	0,081	0,033	0,012	0,984	3,762
0,070	0,256	0,434	0,503	0,481	0,378	0,294	0,224	0,140	0,056	1,230	3,540
0,076	0,254	0,457	0,635	0,559	0,457	0,356	0,229	0,152	0,051	1,476	3,104

 $E_0 = 4.0 \text{ MeV} \quad \psi_0 = 0^\circ$

2,424	0,200	0,058	0,025	0,019	0,011	0,006	0,005	0,003	0,003	0,335	3,635
1,484	0,927	0,343	0,121	0,050	0,064	0,049	0,019	0,015	0,004	0,670	3,218
0,276	0,918	0,897	0,511	0,283	0,241	0,172	0,110	0,055	0,021	1,005	2,879
0,069	0,677	0,709	0,781	0,694	0,538	0,247	0,156	0,062	0,017	1,340	2,420
0,096	0,542	0,829	0,852	0,861	0,669	0,446	0,155	0,066	0,032	1,507	2,126

 $E_0 = 4.0 \text{ MeV} \quad \psi_0 = 27^\circ$

2,354	0,258	0,071	0,043	0,015	0,014	0,009	0,006	0,005	—	0,335	3,596
1,081	0,930	0,479	0,230	0,168	0,098	0,089	0,040	0,035	0,004	0,670	3,177
0,557	0,900	0,728	0,523	0,360	0,249	0,171	0,111	0,060	0,017	1,005	2,728
0,154	0,639	0,947	0,815	0,617	0,463	0,352	0,242	0,132	0,044	1,340	2,273
0,152	0,456	0,659	0,812	0,862	0,761	0,558	0,304	0,101	0,051	1,507	2,121

 $E_0 = 4.0 \text{ MeV} \quad \psi_0 = 45^\circ$

1,986	0,540	0,140	0,060	0,041	0,038	0,029	0,019	0,010	—	0,335	3,500
0,845	1,043	0,551	0,390	0,168	0,108	0,102	0,042	0,030	0,006	0,670	3,052
0,409	0,818	0,818	0,608	0,456	0,327	0,222	0,140	0,070	0,023	1,005	2,571
0,293	0,548	0,731	0,841	0,876	0,655	0,439	0,183	0,073	0,036	1,340	2,129

 $E_0 = 4.0 \text{ MeV} \quad \psi_0 = 63^\circ$

1,202	0,753	0,411	0,244	0,158	0,111	0,073	0,021	0,013	0,009	0,335	3,347
0,493	0,726	0,648	0,631	0,311	0,268	0,207	0,086	0,069	0,009	0,670	2,907
0,321	0,702	0,802	0,702	0,562	0,441	0,321	0,180	0,080	0,020	1,005	2,422
0,102	0,608	0,746	0,813	0,746	0,576	0,407	0,271	0,169	0,068	1,172	2,271

 $E_0 = 4.0 \text{ MeV} \quad \psi_0 = 81^\circ$

0,744	0,767	0,612	0,372	0,333	0,116	0,078	0,070	0,039	0,023	0,335	3,171
0,311	0,796	0,640	0,485	0,427	0,291	0,272	0,233	0,078	0,077	0,670	2,771
0,202	0,755	0,842	0,610	0,465	0,377	0,250	0,203	0,116	0,029	0,837	2,571
0,141	0,470	0,845	1,083	0,563	0,376	0,262	0,235	0,141	0,047	1,005	2,421

$E_c = 2.0 \text{ MeV} \quad \psi = 0^\circ$

4.195	0.561	0.265	0.160	0.068	0.037	0.031	0.021	0.012	0.012	0.166	1.861
1.849	1.892	0.869	0.479	0.315	0.239	0.163	0.098	0.033	0.011	0.333	1.684
1.069	1.897	1.256	0.855	0.641	0.481	0.321	0.214	0.133	0.053	0.500	1.445
0.753	1.255	1.422	1.506	1.338	0.920	0.502	0.251	0.167	0.084	0.667	1.220

 $E_c = 2.0 \text{ MeV} \quad \psi = 27^\circ$

3.665	1.031	0.223	0.204	0.105	0.079	0.053	0.046	—	—	0.166	1.850
1.536	1.942	0.946	0.541	0.381	0.246	0.197	0.098	0.061	0.025	0.333	1.674
0.957	1.641	1.422	1.012	0.739	0.520	0.328	0.191	0.082	0.027	0.500	1.445
0.645	1.197	1.474	1.566	1.381	0.829	0.553	0.276	0.184	0.092	0.667	1.220

 $E_c = 2.0 \text{ MeV} \quad \psi = 45^\circ$

3.170	1.403	0.357	0.256	0.093	0.093	0.047	0.023	0.015	0.008	0.166	1.830
1.253	1.772	1.239	0.749	0.461	0.245	0.173	0.115	0.043	—	0.333	1.656
0.650	1.223	1.376	1.185	0.918	0.650	0.421	0.268	0.153	0.076	0.500	1.445
0.521	1.043	1.303	1.434	1.173	0.762	0.521	0.326	0.130	0.065	0.583	1.370

 $E_c = 2.0 \text{ MeV} \quad \psi = 63^\circ$

2.320	1.521	0.848	0.812	0.195	0.107	0.068	0.068	0.039	0.019	0.166	1.819
1.226	1.328	1.389	0.878	0.572	0.266	0.204	0.164	0.123	0.020	0.333	1.621
0.852	0.939	1.349	1.466	1.173	0.762	0.469	0.235	0.117	0.059	0.500	1.445
0.296	0.888	1.184	1.381	1.184	0.986	0.650	0.395	0.197	0.099	0.583	1.370

 $E_c = 2.0 \text{ MeV} \quad \psi = 81^\circ$

1.957	1.760	0.855	0.559	0.214	0.115	0.082	0.066	0.016	—	0.166	1.778
1.044	1.288	1.323	0.905	0.766	0.452	0.278	0.209	0.070	0.035	0.333	1.570
0.941	1.384	1.384	1.107	0.830	0.603	0.332	0.166	0.111	0.055	0.417	1.445
0.659	1.507	1.601	1.318	0.942	0.659	0.471	0.283	0.188	0.034	0.500	1.295

 $E_c = 1.0 \text{ MeV} \quad \psi = 0^\circ$

7.253	1.813	0.591	0.376	0.228	0.175	0.040	0.040	0.040	0.014	0.077	0.946
3.131	3.256	2.229	1.277	0.676	0.476	0.260	0.175	0.025	0.025	0.154	0.868
1.014	2.683	3.220	2.445	1.491	0.964	0.537	0.298	0.119	0.060	0.231	0.780
0.549	1.978	2.966	2.856	1.978	1.428	0.879	0.549	0.220	0.110	0.269	0.740

 $E_c = 1.0 \text{ MeV} \quad \psi = 27^\circ$

6.927	1.699	0.907	0.360	0.187	0.158	0.144	0.101	0.066	—	0.077	0.946
3.207	3.126	1.976	1.240	0.916	0.431	0.350	0.243	0.081	0.027	0.154	0.863
1.552	2.858	2.613	1.960	1.470	0.980	0.653	0.408	0.245	0.032	0.231	0.780
0.932	2.329	2.950	2.950	2.174	1.242	0.776	0.466	0.311	0.155	0.269	0.700

 $E_c = 1.0 \text{ MeV} \quad \psi = 45^\circ$

6.145	2.599	0.930	0.361	0.275	0.172	0.086	0.052	0.017	—	0.077	0.940
2.777	3.859	1.954	1.508	1.028	0.651	0.171	0.137	0.034	0.034	0.154	0.858
0.583	3.164	2.997	2.165	1.582	1.082	0.666	0.333	0.166	0.083	0.231	0.780
1.513	2.185	2.521	2.521	2.185	1.513	0.840	0.504	0.336	0.168	0.269	0.700

 $E_c = 1.0 \text{ MeV} \quad \psi = 63^\circ$

5.570	2.500	1.425	0.548	0.417	0.154	0.110	0.044	0.022	0.022	0.077	0.925
2.269	3.647	1.913	1.879	1.112	0.712	0.400	0.356	0.069	—	0.154	0.842
1.129	2.635	3.287	1.882	1.204	0.828	0.602	0.376	0.226	0.075	0.192	0.820
0.889	1.904	2.539	2.412	1.777	1.269	0.888	0.635	0.381	0.127	0.231	0.780

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 $E = 1.0 \text{ MeV}$ $\psi_0 = 81^\circ$

3.923	3.047	2.129	0.918	0.668	0.167	0.167	0.125	0.042	—	0.077	0.844
2.013	3.245	1.907	1.695	1.271	1.060	0.848	0.424	0.22	0.106	0.154	0.780
1.053	2.457	3.159	2.457	1.580	1.053	0.702	0.527	0.351	0.175	0.192	0.740
1.554	3.885	3.497	2.331	1.165	0.777	0.777	0.389	0.388	0.388	0.231	0.660

 $E_0 = 0.7 \text{ MeV}$ $\psi_0 = 0^\circ$

10.811	2.178	1.001	0.530	0.177	0.118	0.098	0.039	0.020	—	0.049	0.668
4.533	4.457	2.871	1.473	0.944	0.680	0.642	0.302	0.151	0.076	0.099	0.620
2.389	3.867	3.640	2.843	2.047	1.365	0.910	0.455	0.227	0.114	0.149	0.560
1.662	3.087	3.562	3.562	2.850	1.900	1.188	0.475	0.238	0.237	0.174	0.533

 $E_0 = 0.7 \text{ MeV}$ $\psi_0 = 27^\circ$

9.504	3.347	0.987	0.515	0.215	0.193	0.150	0.107	0.043	—	0.049	0.664
5.151	3.920	2.917	1.504	0.912	0.722	0.592	0.319	0.091	0.046	0.099	0.618
1.636	3.817	5.180	2.863	1.772	1.050	0.682	0.409	0.273	0.136	0.149	0.560
1.001	3.252	5.253	3.502	2.251	1.501	1.001	0.500	0.250	0.250	0.174	0.533

 $E_0 = 0.7 \text{ MeV}$ $\psi_0 = 45^\circ$

9.520	3.134	1.155	0.636	0.448	0.118	0.094	0.047	—	—	0.049	0.660
3.394	5.510	2.607	2.115	1.131	0.541	0.492	0.393	0.049	—	0.099	0.618
2.511	4.752	3.855	2.331	1.524	0.897	0.539	0.359	0.179	0.090	0.124	0.587
2.252	3.700	3.378	2.574	2.001	1.609	1.126	0.644	0.322	0.161	0.149	0.560

 $E_0 = 0.7 \text{ MeV}$ $\psi_0 = 63^\circ$

7.686	3.727	1.970	0.533	0.394	0.182	0.152	0.121	0.020	—	0.049	0.664
3.040	3.686	3.234	2.587	1.487	1.423	0.712	0.323	0.065	—	0.099	0.604
1.715	4.230	3.659	2.744	1.944	1.258	0.686	0.457	0.229	0.114	0.124	0.587
1.649	3.505	3.917	3.505	2.268	1.649	1.031	0.619	0.412	0.206	0.149	0.533

 $E_0 = 0.7 \text{ MeV}$ $\psi_0 = 81^\circ$

6.788	4.349	2.227	1.220	0.371	0.212	0.212	0.053	—	—	0.049	0.648
4.883	4.658	3.207	1.650	0.677	0.524	0.458	0.153	—	—	0.074	0.612
3.401	4.457	3.753	2.550	1.173	0.704	0.704	0.284	0.117	0.117	0.069	0.580
1.180	4.167	4.865	3.175	1.924	1.389	0.784	0.327	0.169	0.168	0.124	0.530

 $E_0 = 0.4 \text{ MeV}$ $\psi_0 = 0^\circ$

19.056	8.840	1.500	0.769	0.386	0.329	0.373	0.073	0.037	—	0.023	0.384
8.531	7.121	4.301	2.961	1.622	1.622	0.916	0.282	0.282	0.141	0.047	0.360
3.870	7.703	7.944	3.852	2.407	1.855	1.204	0.722	0.482	0.481	0.070	0.325
3.280	7.431	6.900	5.845	3.282	1.645	1.224	0.822	0.622	0.411	0.082	0.320

 $E_0 = 0.4 \text{ MeV}$ $\psi_0 = 27^\circ$

17.067	5.002	1.729	1.057	0.625	0.184	0.147	0.110	0.074	0.037	0.023	0.334
8.553	8.403	3.735	2.501	1.837	1.478	0.309	0.233	0.156	0.156	0.047	0.360
3.940	6.106	6.784	5.654	3.832	2.035	1.557	0.905	0.452	0.223	0.070	0.335
3.404	4.808	6.410	6.811	4.407	2.404	1.603	1.202	0.801	0.401	0.082	0.320

 $E_0 = 0.4 \text{ MeV}$ $\psi_0 = 45^\circ$

15.923	5.667	2.423	0.910	0.540	0.435	0.135	0.020	0.045	0.045	0.023	0.384
6.861	7.933	4.410	2.940	2.352	1.470	0.734	0.450	0.330	0.204	0.047	0.358
4.537	6.825	6.334	4.460	2.937	1.788	1.011	0.511	0.255	0.128	0.050	0.345
3.875	6.412	7.076	4.422	3.008	2.211	1.769	1.105	0.663	0.221	0.070	0.320

 $E_0 = 0.4 \text{ MeV}$ $\psi_0 = 63^\circ$

13.650	6.883	2.776	1.215	0.578	0.520	0.347	0.231	0.116	—	0.023	0.360
6.141	7.788	5.773	3.562	2.702	1.228	0.408	0.328	0.246	0.123	0.047	0.354
3.221	5.981	6.288	4.448	3.221	2.454	1.667	1.073	0.460	0.153	0.050	0.345
2.864	5.820	6.207	6.207	4.433	2.364	1.162	0.887	0.531	0.296	0.070	0.325

 $E_0 = 0.4 \text{ MeV}$ $\psi_0 = 81^\circ$

13.853	6.299	3.695	1.764	0.840	0.252	0.168	0.084	—	—	0.023	0.372
7.783	7.665	5.687	2.102	1.854	1.113	0.742	0.371	—	—	0.035	0.366
6.001	7.720	6.219	4.289	1.287	1.072	1.072	0.258	0.214	—	0.047	0.348
4.766	6.271	5.770	4.515	3.010	2.259	1.505	0.603	0.502	0.251	0.069	0.335

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